

# “Consumer Behavior”

Reynolds

- I. Neoclassical economics assumes that individuals are “rational.”
  - A. Rational requires that individuals know their objective and all feasible alternatives.
  - B. Individuals must establish criteria to evaluate each alternative with respect to the objective.
  - C. Principle agent problem
    1. When an agent acts on behalf of a principal, there may be imperfect information between the principal and agent.
    2. The objectives of the agent may not be consistent with those of the principal
    3. the principal-agent problem can be viewed as the process by which a contract can be designed to motivate the agent to act in the principal’s interests.
- II. Consumer choice is dependent on the set of possibilities and the preferences of the individual.

- A. The set of possible choices is defined by;
  1. The income or budget of the individual
  2. The prices of the set of goods (and services) available and relevant to the individual
  3. The budget constraint can be defined as:

$$M \geq P_X Q_X + P_Y Q_Y + \dots + P_N Q_N$$

To simplify your life, consider two goods rather than N goods:

$$M \geq P_X Q_X + P_Y Q_Y$$

Where:

M = income

$P_X$  = Price of good X

$Q_X$  = Quantity of good X

$P_Y$  = Price of good Y

$Q_Y$  =Quantity of good Y

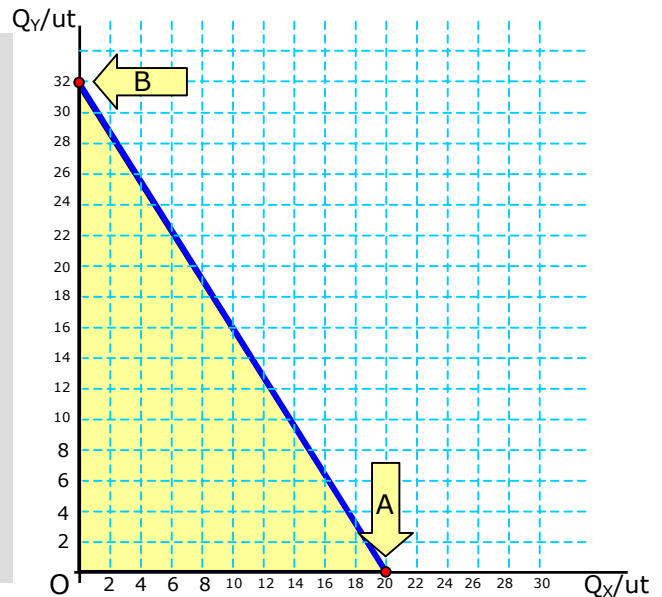
If income is \$80 per month ( $M=80/m$ ) while the price of good X is \$4 ( $P_X=4$ ) and the price of good Y is \$2.50 ( $P_Y=2.50$ ), the budget constraint can be graphed, If the entire \$80 is spent on good X, 20 units can be purchased. This is shown as point A on the graph to the right,  $Q_X = 20$

If the \$80 is spent on good Y, the maximum amount of good Y is 32,  $Q_Y = 32$ . This is shown as point B.

Any combination of goods X and Y (called a “bundle or market basket”) that falls on the line AB requires an expenditure of the entire \$80. For example, a purchase of  $Q_Y=16$  and  $Q_X = 10$  requires an expenditure of \$80. This is the **budget constraint** or **budget line**. All combinations on or inside the constraint are **feasible**.

Any combination that lies in the triangle OAB (in yellow) is feasible and requires an expenditure less than \$80.

Any combination that lies outside the line AB will cost more than \$80. This is the infeasible set.



## Problems:

1. If  $Q_X = 16$ , How many units of good Y can be purchased?
2. If  $Q_Y = 24$ , How many units of good X can be purchased?

4. Note that the quantity and income is regarded as a "flow," i.e. it is measured as an amount during a specific period of time.
5. Summary:
- a) Bundles or market baskets that lie on the budget line or budget constraint AB are feasible and require an expenditure of exactly \$M.
  - b) Bundles that line inside the budget line cost less than \$M and are also feasible.
  - c) Bundles that lie outside the budget constraint require an expenditure of more than \$M and are infeasible or unaffordable.
6. Relative Prices and the slope of the budget constraint

The slope of the line AB can be described as "rise over run," which would be  $-\frac{B}{A} = -\frac{32}{20} = -\frac{4}{2.5} = -1.6$

Note that the X intercept ( $Q_x=20$ ) was calculated by dividing the budget (M) by the price of good X ( $P_x$ ),

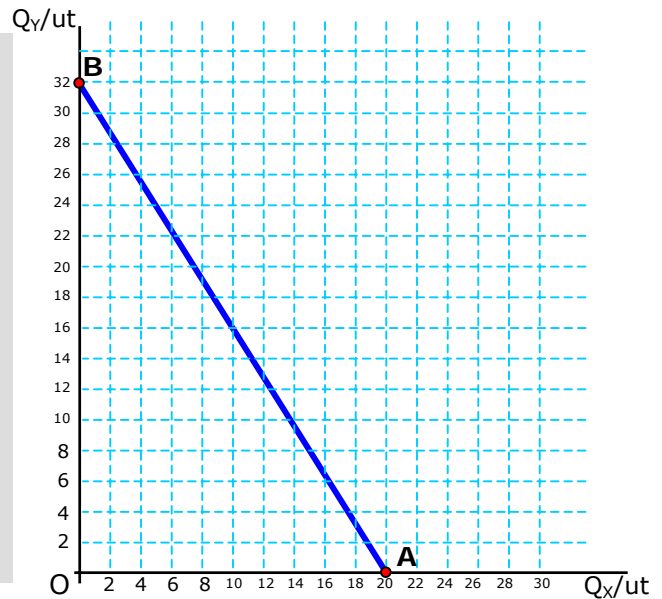
$$Q_{Xintercept} = \frac{M}{P_x} = \text{"run"}$$

The value of the Y intercept was calculated by the budget (M) divided by the price of good Y ( $P_y$ ),

$$Q_{Yintercept} = \frac{M}{P_y}, \text{ so the "rise"}$$

$$\text{from point B to A is } -\frac{M}{P_y}$$

This makes the slope of the budget constraint  $-\frac{P_x}{P_y}$



7. The equation for the budget constraint can be expressed:

$$a) M = P_x Q_x + P_y Q_y$$

$$b) Q_y = \frac{M}{P_y} - \frac{P_x}{P_y} Q_x$$

8. A change in income ( $\Delta M$ ) is shown by a parallel shift of the budget constraint.

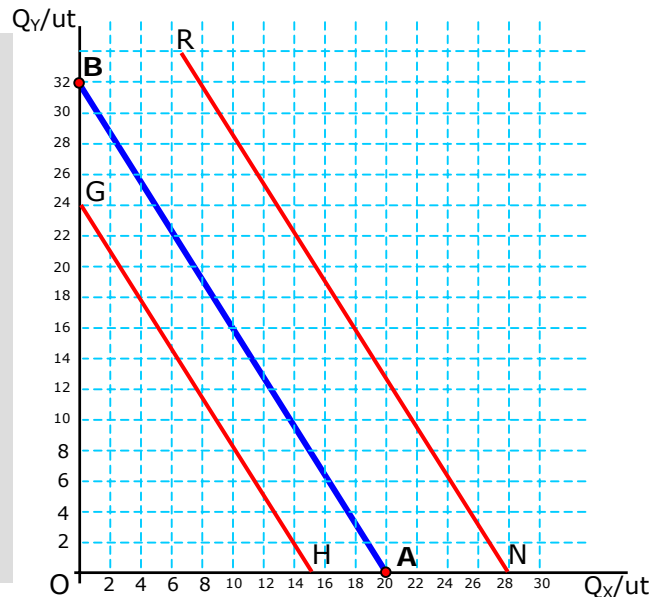
When  $M = \$80$ ,  $P_x = \$4$  and  $P_y = \$2.50$  the budget constraint is AB.

If M falls to  $M' = \$60$  ( $\Delta M = -20$ , and prices are unchanged) the new budget line becomes line GH. A decrease in the budget shifts the budget constraint to

the left. The  $Q_y$  intercept (point G) will be  $\frac{60}{2.5} = 24$ , the

$$Q_x \text{ (point H) intercept is } \frac{M}{P_x} = \frac{60}{4} = 15$$

An increase in the budget (with prices unchanged) will shift the budget to the right. If the income or budget increases to \$112 ( $\Delta M = 32$ ) the budget constraint will shift to line RN.



9. A change in relative prices can be shown by a rotation of the budget line (i.e. its slope will change)

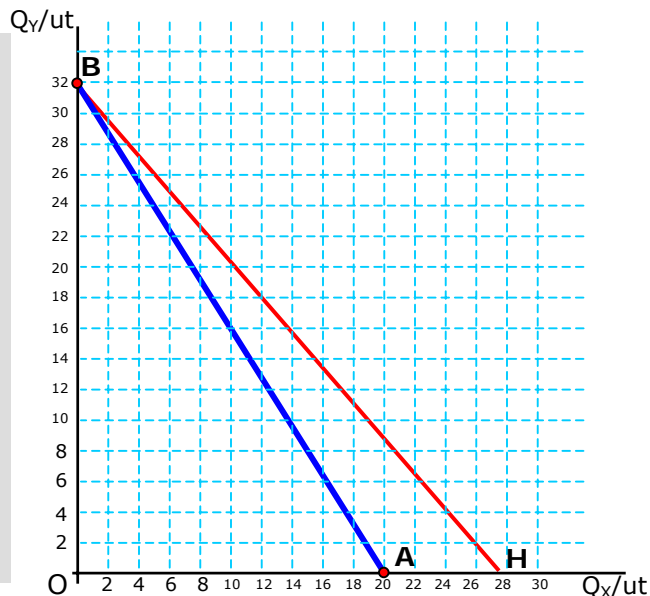
Given  $M = \$80$ ,  $P_X = \$4$  and  $P_Y = \$2.50$  the budget constraint is shown by line AB. If the price of X ( $P_X$ ) decreases the budget constraint will rotate outward along the X-axis. If the price of X changes ( $\Delta P_X = -1$ , so  $P_X' = \$3$ ), the new  $Q_X'$  intercept will be

$$Q_{X\text{-intercept}}' = \frac{80}{3} = 26.67$$

The new budget constraint (given the new  $P_X'$ ) is represented by the line BH.

Note that income ( $M$ ) remained at  $\$80$  and  $P_Y$  is still  $\$2.50$ . The reduction in the price of X changed the  $Q_X$ -intercept.

While the income ( $M$ ) remains at  $\$80$ , the reduction in the price of good X ( $P_X$ ) has rotated out, thereby increasing the set of bundles that can be purchased with the original  $\$80$ . The nominal income remains constant but there is an increase in "real income."



10. An increase in the price of good X ( $\Delta P_X > 0$ ) will rotate the budget constraint in along the X-axis. Let the price of good X increase to  $\$5$ . What is the new  $Q_X$  intercept? Draw in the new budget constraint. What has happened to the feasible set of bundles that can be purchased?

11. If the price of good Y ( $P_Y$ ) changes while income ( $M$ ) and the price of good X ( $P_X$ ) remain constant, the budget constraint will rotate along the  $Q_Y$  axis. An increase in the price of good Y will rotate the budget constraint in along the axis while a decrease will rotate it out. Raise the price of good Y to  $P_Y' = \$3$  while  $M$  and  $P_X$  are unchanged. What is the new  $Q_Y'$ ? What will happen if the  $\Delta P_Y < 0$ ?

**Problems:**

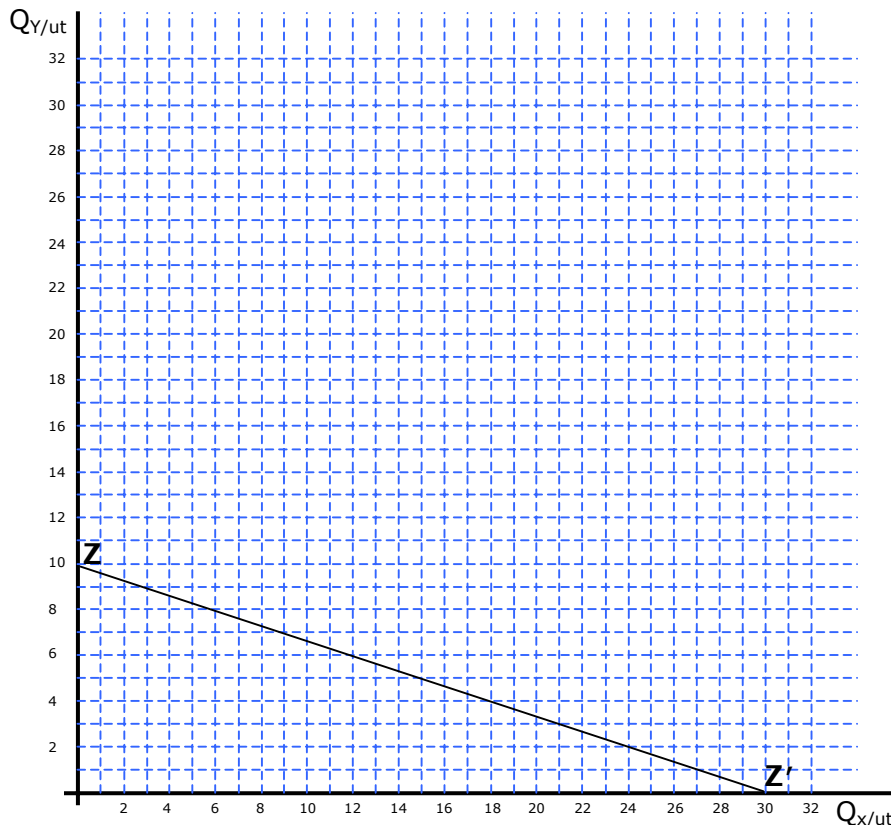
1) Given an income of  $\$120$ ,  $P_X = \$5$  and  $P_Y = \$10$ , construct the budget constraint.

2) If the price of X ( $P_X$ ) increases to  $\$8$  ( $M = 120$ ,  $P_Y = 10$ ), draw the new budget constraint.

3) If the income increases to  $\$240$  ( $M'$ ) given  $P_X' = 8$  and  $P_Y = 10$ , construct the budget constraint.

4) With the income ( $M' = 240$ ) and  $P_X' = 8$ , Show an increase in the price of Y ( $P_Y' = 12$ ) to  $\$12$ .

5) Given the budget constraint ZZ' is when  $M = \$400$ , what are the prices of goods X and Y?



12. Budgets with more than two goods are in multidimensional space and are represented by a hyperplane. Three goods are in three-dimensional space, n-goods in n dimensional space.
13. Pricing strategies such as quantity discounts, deductibles and co-payments may result in "kinked" budget constraints.

### III. Consumer Preferences

A. Consumers are said to have a utility function, i.e. the satisfaction or utility that an individual has is dependent on a set of independent variables,  
 $U_i = f(Q_X, Q_Y, \dots, Q_N, \text{Health, Attitude, etc...})$

Since health, attitude and that sort of thing are difficult if not impossible to measure, neoclassical economics focuses on the quantities of goods and services ( $Q_X, Q_Y, Q_N$ ).

B. There are two approaches to explaining consumer choices (or behavior), one is with "marginal utility" analysis which requires cardinal measurement of utility. Cardinal utility measurement requires the agent to measure their satisfaction in numeric terms. A pizza gives me 16 utils of satisfaction and a tofu burger gives me 8 utils, i.e. I like pizza twice as much as I like tofu burgers. The second approach is based on ordinal utility preferences. This approach presumes that the agent can order their preferences, i.e. I like pizza more than I like tofu burgers. This approach is referred to as "indifference analysis."

C. Indifference analysis presumes the agent can always identify their preferences. For any two bundles of goods (A and B) the consumer must be able to rank or give a preference ordering to all potential bundles of goods:

1. I prefer A to B, i.e. A gives me more satisfaction than B, or
2. I prefer B to A, or
3. I am indifferent between A and B, i.e. A and B give me the same satisfaction.

D. Conditions required for indifference analysis

1. **Completeness** – All combinations or bundles of goods can be rank ordered. Clearly, lack of information about feasible alternatives can lead to problems.
2. **More is preferred to less** – assumes that individuals derive utility from the good. Note that many goods may yield negative utility at some point. If goods have negative utility or disutility the indifference curves will not be "normally" shaped.
3. Agents must maintain the conditions of **transitivity**. If you prefer hamburgers to hotdogs and hotdogs are preferred to tofu burgers then you must prefer hamburgers to tofu burgers.
4. Frank argues that "**convexity**" is also required. This requirement is only to insure the solution for utility maximization is determinant. It is possible that there may not be a single solution to optimize a function.

E. Indifference curves are like a topological or contour map. While the contour map shows the topological features of a geographic area the indifference curves and indifference map shows the characteristics of an individual's utility function. In Figure 1 there is a contour map. Each contour line represents an elevation. The lines are constructed at 500-foot elevation differences. The outside line represents all points that have an elevation of 500'. The second line has an elevation of 1000', the third line an elevation of 1500' and so forth. The contour lines represent a molehill with an elevation of something over 2500' and less than 3000'. Do not make a mountain out of a molehill. An indifference map shows the features of an individual's preferences. Consider an indifference map for two goods (an indifference map for n goods will be in n-dimensional space),  $Q_X$  and  $Q_Y$ . The individual is endowed with  $Q_{XA}=12$  and  $Q_{YA}=10$ . This is shown in Figure 2.

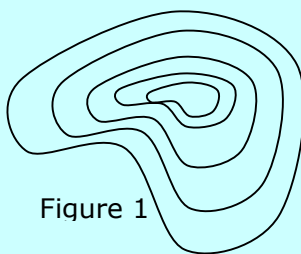


Figure 1

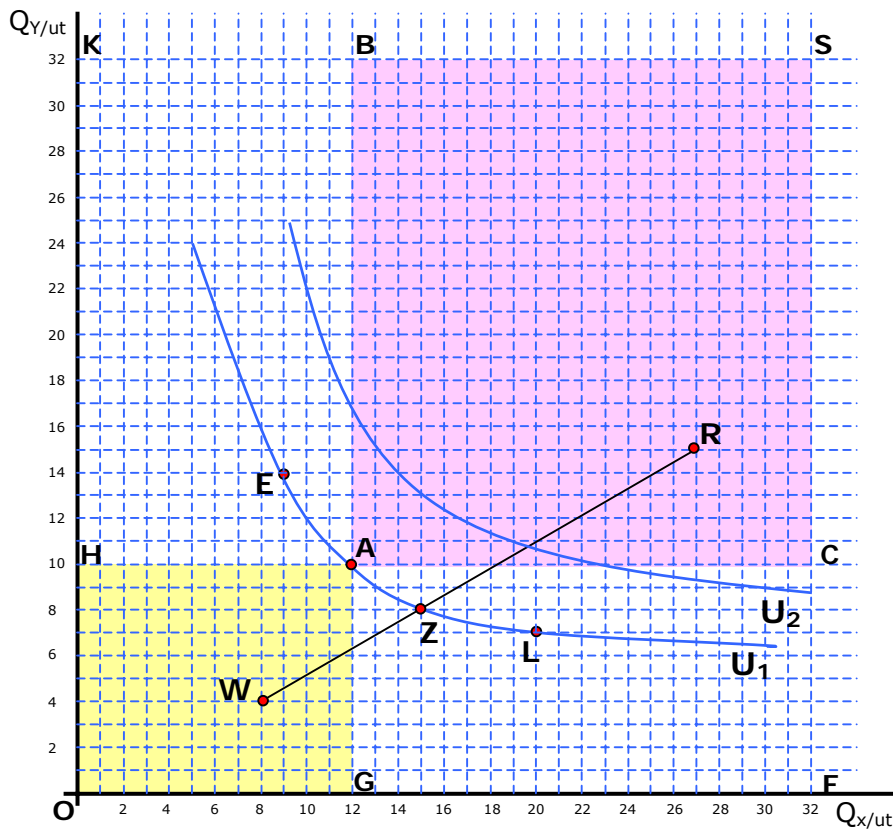


Figure 2

In Figure 2 the individual has  $Q_{XA} = 12$  and  $Q_{YA} = 10$ . Any bundle in area OGAH is less preferred than the bundle at A. All bundles in the area OGAH has less of good X or good Y or less of both good X and Y than at bundle A. Since more is preferred to less any combination of goods in the area OGAH is less preferred.

All bundles in area ACSB are preferred to the bundle at point A because there is more of good X or Y or both than at point A.

The bundles in the areas HABK and GFCA represent bundles that may yield more, less or the same utility as the bundle at point A. Frank reasons that the bundle at A is preferred to that at point W. The bundle at R is preferred to that at A so along the line WR there is one bundle that is equally preferred to the bundle at A.

Bundles L and E can be found using similar reasoning. Bundles at E, A, Z and L (and all other bundles found on curve  $U_1$ ) are equally preferred. The indifference curve passes through all combinations of good X and Y that yield the same level of satisfaction of utility. Indifference curve  $U_0$  represents another set of bundles that yield a higher level of utility than those bundles on  $U_1$ . It is not possible for one bundle to yield two levels of utility to the same person; therefore the indifference curves cannot touch. If the conditions of transitivity are met, the curves cannot intersect.

In Figure 3, an indifference curve ( $U_1$ ) is shown. Notice that this is not the normal case. There are sections of the curve where "more is not preferred to less." If the individual has more of good Y than  $Q_{Yb}$  (18) and only  $Q_{Xb}$  (3) units of good X (at point b), additional units of Y have negative utility (the MU of additional units of Y are less than 0). Notice that as the units of Y increases to  $Q_{Yc}$  (holding  $Q_{Xb}$  constant) the individual is moved to a lower indifference curve ( $U_0$ ). The positive slope of the indifference curve in this region shows

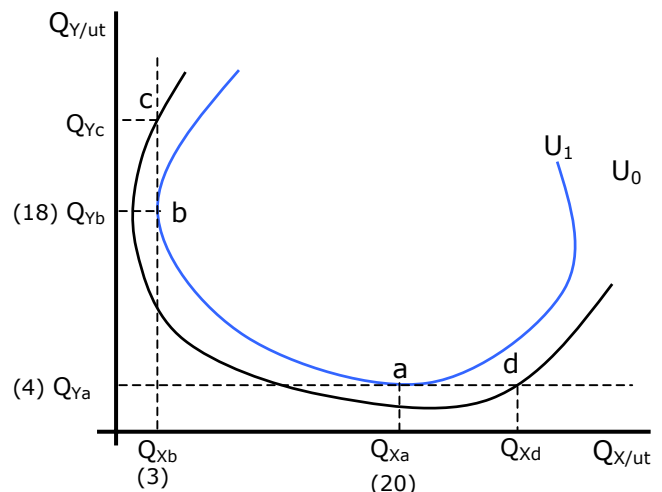


Figure 3

that additional units of Y require more of good X to offset the reduction of utility that results from additional units of Y. It would not be "rational" for an agent to consume more of good Y if it reduced utility. For this reason "normal" indifference curves are shown in the region where they are negatively sloped (and are usually convex to the origin).

F. **Marginal rate of Substitution** ( $MRS_{XY}$ )

The indifference curve shown in Figure 4 is "normally shaped"

1. it is downward sloping
2. it is convex

Given an individual with 13 units of good Y ( $Q_{Ya}=13$ ) and 4 units of good X ( $Q_{Xa}=4$ ) at point a, they attain  $U_1$  level of utility. They have the same level of utility at point b (it is on the same indifference curve). Therefore, the agent is willing to trade 3 units of good Y for 1 unit of good X. The slope of the indifference curve between points a and b is a measure of the "willingness to trade" one good for the other and is called the

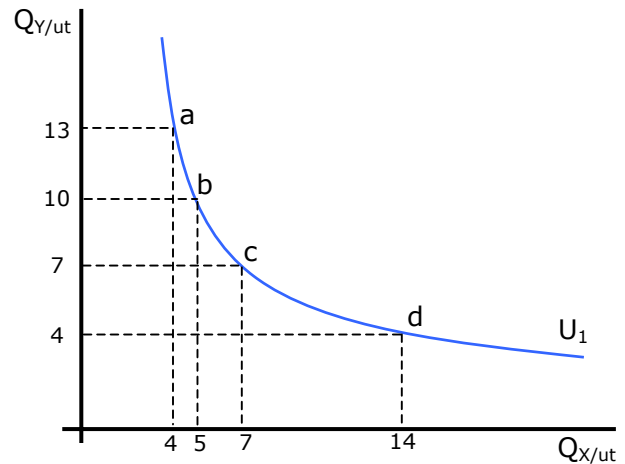


Figure 4

**Marginal Rate of Substitution of good X for Y** ( $MRS_{XY}$ ). Typically, the MRS will be stated as the absolute value of the slope of the indifference curve between point a and b ( $MRS_{XYa}$ ). It is the rate at which the individual is willing to trade good X for good Y and maintain the same level of utility.

$$MRS_{XY} = \left| \frac{\Delta Q_Y}{\Delta Q_X} \right| \approx \frac{dy}{dx} \approx \frac{\partial Y}{\partial X}$$

When the  $MRS_{XY}$  is calculated by:  $MRS_{XY} = \frac{\Delta Q_Y}{\Delta Q_X}$ , it is the slope of the arc between two points on an indifference curve. When the  $MRS_{XY}$  is calculated by:  $MRS_{XY} = \frac{dQ_Y}{dQ_X}$ , it is the slope of a tangent to the indifference curve at a point.

G. Diminishing MRS

As an individual has more and more of a good, its marginal utility (MU) declines relative to the marginal utility of other goods. In Figure 4 as the individual obtains more and more of good X and less and less good Y the  $MU_X$  declines relative to the  $MU_Y$ . At point a the individual is willing to trade 3 units of good Y ( $\Delta Q_Y = 3$ ) for one unit of good X ( $\Delta Q_X = 1$ ). This is represented by a move from point a to point b. The  $MRS_{XYa}$  (marginal rate of substitution of a trade from point a to point b) is:

$$MRS_{XYa} = \left| \frac{\Delta Q_Y}{\Delta Q_X} \right| = \left| \frac{-3}{1} \right| = |-3|$$

To maintain the same level of utility (stay on the same indifference curve,  $U_1$ ) and move from point b to point c the individual is willing to trade 3 units of Y ( $\Delta Q_Y = 3$ ) for 2 units of X ( $\Delta Q_X = 2$ ). The MRS is 1.5.

As the individual has more and more X, they are willing to trade more and more X for the same (or less) of good Y. This is "diminishing MRS" and

is the same thing as the “convexity” requirement of normal indifference curves.

H. Special cases of Indifference Curves

1. **When good Y has no utility** to the individual and X has positive utility the indifference curves are represented by vertical lines as shown in Figure 5. As the individual acquires more X ( $Q_{X1}, Q_{X2}, \dots$ ) they move to higher indifference curves ( $U_1, U_2, \dots$ ). More of good Y with the same amount of good X (move from point a to d as the quantity of Y increases from  $Q_{Y1}$  to  $Q_{Y2}$ ) leaves the individual with the same level of utility ( $U_1$ ). If good X has no utility and good Y has positive utility the indifference curves will be horizontal. More Y will move the individual to higher indifference curves while more X leaves them on the same indifference curve.

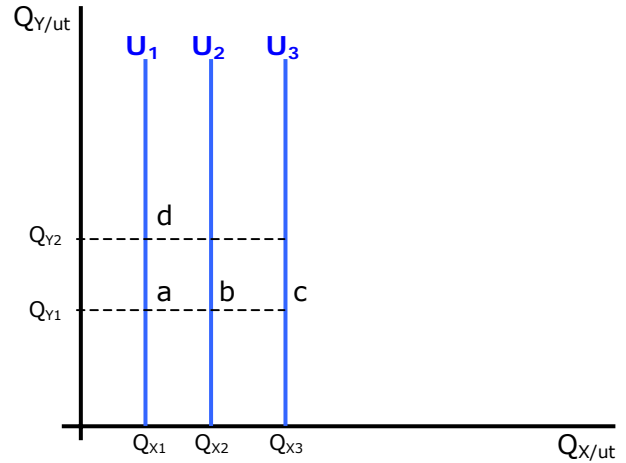


Figure 5

2. **Substitute goods** are represented in Figure 6. The individual is indifferent (has the same level of utility,  $U_2$ ) at all points on the indifference curve,  $U_2$ . The agent is equally satisfied at points, a, b and c. The quantity  $Q_{Y2}$  is equally preferred as quantity  $Q_{X3}$ . The individual perceives the two goods as perfect substitutes.

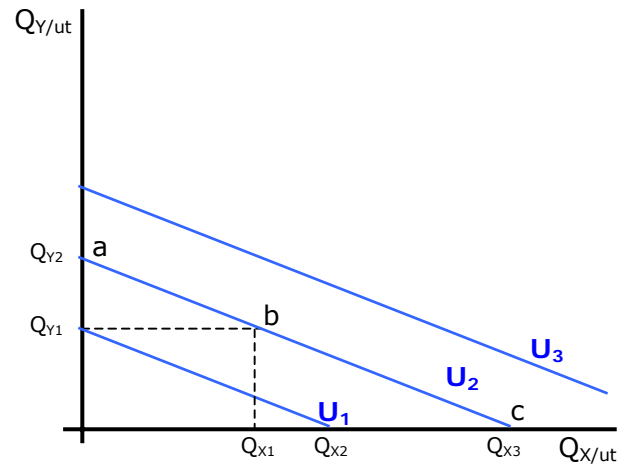


Figure 6

3. **Complementary goods** will result in indifference curves that are right angles. If goods X and Z are perfect complements the indifference curves can be represented in Figure 7. At point A the individual has  $Q_X=2$  and  $Q_Y=1$ . They always use 2 units of X for every one unit Y. They never use any X unless they have twice as much Y. If they are given two more units of X but no more Y (represented as a move to the bundle at point d) they have the same level of satisfaction as at point a. Similarly, if they have 2 units of X and 1 of Y at point a, an increase in the amount of Y to 2 (at point c) will not increase their level of satisfaction. If the individual is given 2 more X's and one more Y (a move to b), they will be on a higher indifference curve,

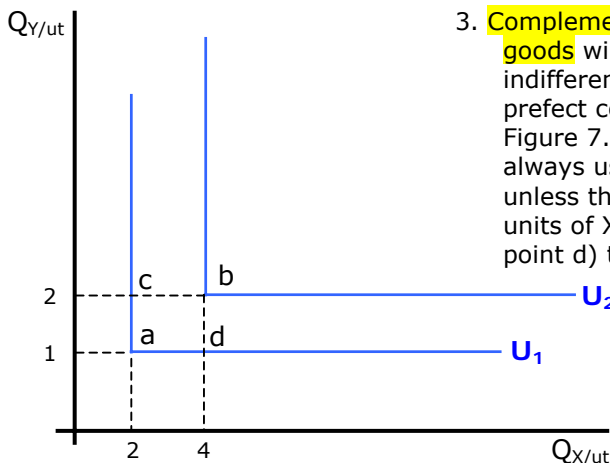


Figure 7

#### IV. Constrained Optimization of Utility

In Figure A an individual with a known utility function (indifference curves  $U_1$  and  $U_2$  are shown) has an income ( $M$ ) and is confronted with a set of relative prices for goods X and Y,  $P_X$  and  $P_Y$  respectively. The budget constraint can be calculated and graphed. Remember that the  $Q_X$  intercept will be  $\frac{M}{P_X}$ . The  $Q_Y$  intercept will be  $\frac{M}{P_Y}$ . The budget constraint is a straight line between the two intercepts and includes point a, b, and c (as well as an infinite set of other bundles). Any point on the budget constraint is affordable or feasible given  $M$ ,  $P_X$  and  $P_Y$ . The individual can buy any bundle of goods along the budget constraint. If the purchase  $Q_{Xa}$  and  $Q_{Ya}$  at point a or  $Q_{Xc}$  and  $Q_{Yc}$  at point c, they will obtain  $U_1$  level of satisfaction. However, they would like to maximize their utility give the income and prices of the goods. The highest indifference curve (level of utility) they can reach will be at point b (purchase  $Q_{Xb}$  and  $Q_{Yb}$ ).

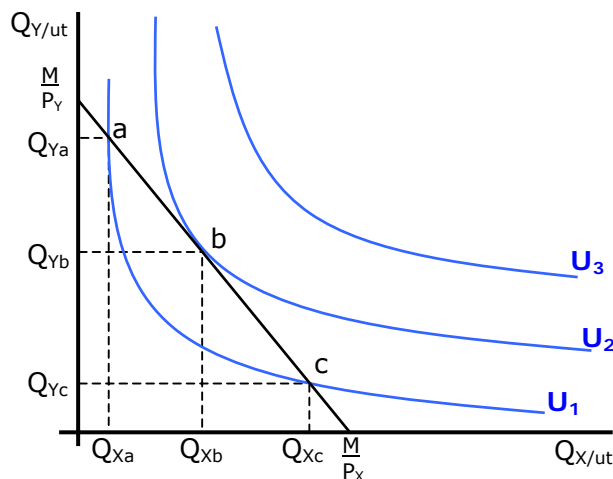


Figure 8

Note that the budget constraint is tangent to the highest indifference curve the individual can reach, the  $MRS_{XY} = \frac{P_X}{P_Y}$ . The slope of the budget line  $\left(\frac{P_X}{P_Y}\right)$  is a measure of the rate at which good X can be exchanged for good Y. The  $MRS_{XY}$  is a measure of rate at which the individual is willing to exchange good X for good Y. When what the individual is willing to do is the same as what they can do, they are in equilibrium, i.e. there is no reason to alter their behavior.

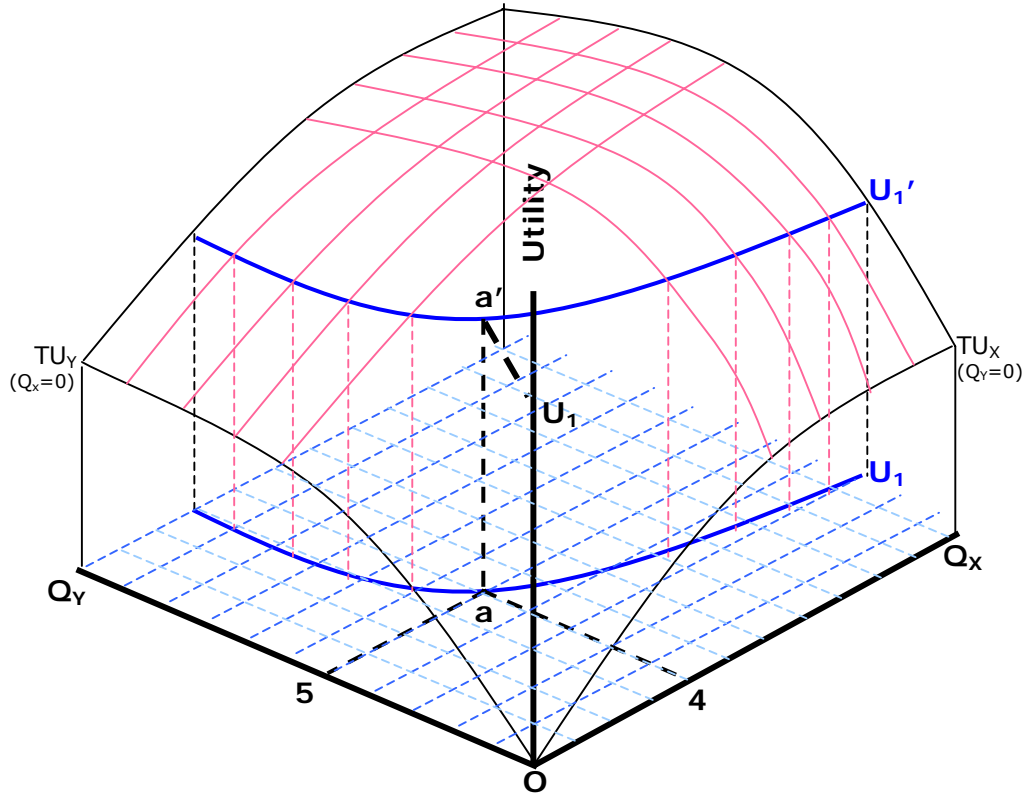
#### V. The "Numeraire" or composite good

Two-dimensional models of indifference curves are limited to two goods. A useful convention is to consider one of the two goods as a composite or numeraire good. A numeraire good is a bundle that represents all other goods. Money can be used as a numeraire good since it is fungible. A dollar (pound or euro) will buy a dollar's (pound's or euro's) worth of other goods.

#### VI. Construction of an indifference map

A. An indifference map is really a three dimensional model presented in two dimensions. The map includes good X, good Y and utility. Using the metaphor of the contour map remember that a topological map presents three dimensions on a flat piece of paper, north-south, east-west and elevation. The construction of an indifference map is shown in Figure 9.

Figure 9



An indifference curve is a two-dimensional model of a three-dimensional concept. It is like a topological map and is constructed by projecting a level of utility (or elevation) onto a flat (two-dimensional surface).

In the graph above the quantity of good X ( $Q_x$ ) is on the right axis while the quantity of good Y ( $Q_y$ ) is on the left axis. The level of utility (elevation) is measured on the vertical axis.

In the example we have 4 units of good X ( $Q_x=4$ ) and 5 units of good Y ( $Q_y=5$ ). These two coordinates locate the point a on the graph. The "elevation" or level of utility ( $U_1$ ) at that point is a' on the utility surface above.

There are an infinite number of points on the utility surface that are at the same elevation. A line through these points is a "contour line," labeled  $U_1'$ . The contour line,  $U_1'$  can be projected down to the surface of the graph to show all combinations of good X and Y that yield the level of utility  $U_1$ .

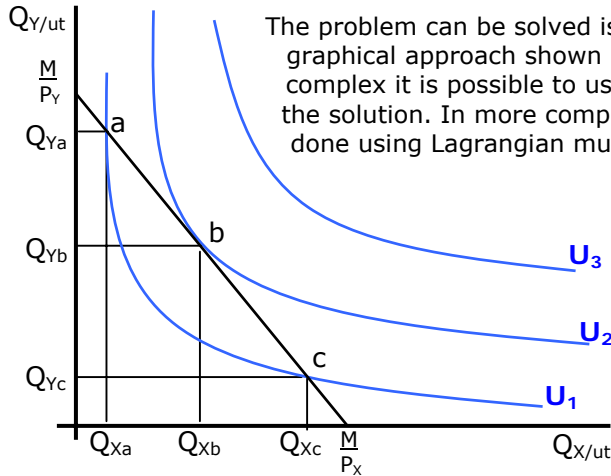
There are an infinite number of elevations on the utility surface that result in an infinite number of contour lines that do not intersect. The projections of these contour lines result in an infinite number of indifference curves that are "every where dense," do not intersect and are convex to the origin.

## VII. Mathematics of constrained utility maximization and consumer equilibrium

In Figure 8 (above) the graphics of utility maximization were shown by a tangency of the budget constraint and the indifference curve. The optimal bundle of goods that a consumer can purchase given income, prices and preferences can be shown mathematically. It is also useful to understand the conditions that result in consumer equilibrium.

The conditions required include:

- a) The agent has a given income (M)
- b) The agent buys two goods ( $Q_X$  and  $Q_Y$ ), one may be a numeraire or composite good
- c) Both goods have positive utility (indifference curves are negatively sloped and higher indifference curves show higher levels of utility, "more is preferred to less")
- d) The agent faces a set of prices (generally it is assumed that these prices are determined by market forces),  $P_X$  and  $P_Y$ .
- e) The agent is a utility maximizer
- f) The agent can rank order (ordinally rank) all bundles of goods
- g) Agents preferences are transitive
- h) All commodities are perfectly divisible



The problem can be solved in several ways that are consistent with the graphical approach shown in Figure 8. If the utility function is not very complex it is possible to use a little algebra and simple derivatives to find the solution. In more complex problems, constrained optimization can be done using Lagrangian multipliers.

**Problem:**

Given:

a utility function  $U_{XY} = X^1Y^1$

$P_X = \$4$

$P_Y = \$6$

$M = \$100$

Find the  $Q_X$  and  $Q_Y$  that will maximize Utility given  $M$ ,  $P_X$ ,  $P_Y$  and preferences.

Figure 8