

"Demand and Consumer Behavior"

Reynolds

I. Individual's Demand Function

An individual's demand function is a representation of the equilibrium conditions for a buyer. A market demand function is a model of the equilibrium conditions for a set of buyers.

A. Individual Demand Function

Each individual has a preference function and income or budget. They are presumed to maximize their utility subject to a set of relative prices, their preferences and income. Any change in prices, relative prices, income or preferences may alter the individual's behavior. Indifference curves and budget constraints can be used to explain how the individual reacts to each possible change in circumstances.

a) This individual's demand function can be stated:

$$Q_x = f(P_x, P_y, M, \text{preferences}, \dots)$$

Q_x is normally the dependent variable. Its value is a function of the values of the other independent variables. It is useful to identify the effect that a change in any independent variable has on the value of Q_x .

b) A change in P_x results in a "change in quantity demanded." This is shown as a movement along a demand curve.

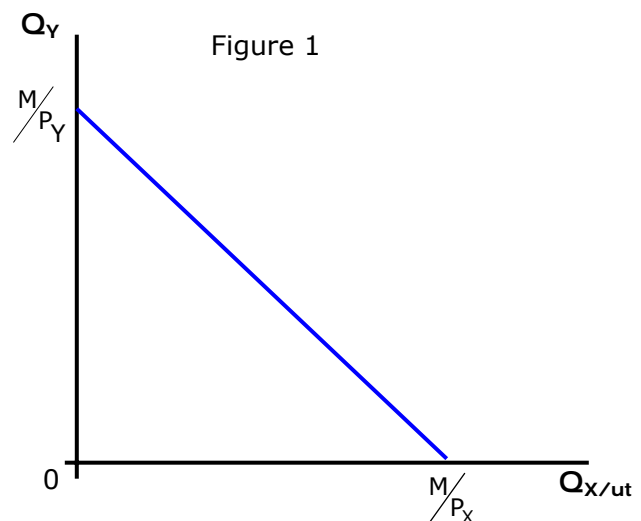
c) A change in P_y , M or preferences will cause a "change in demand" or a shift of demand. It can be thought of as a movement of the demand curve.

d) If P_y , M and preferences are unchanged, the demand curve or schedule can be stated as:

$$Q_x = f(P_x), \text{ ceteris paribus}$$

B. Construction of individual's demand function

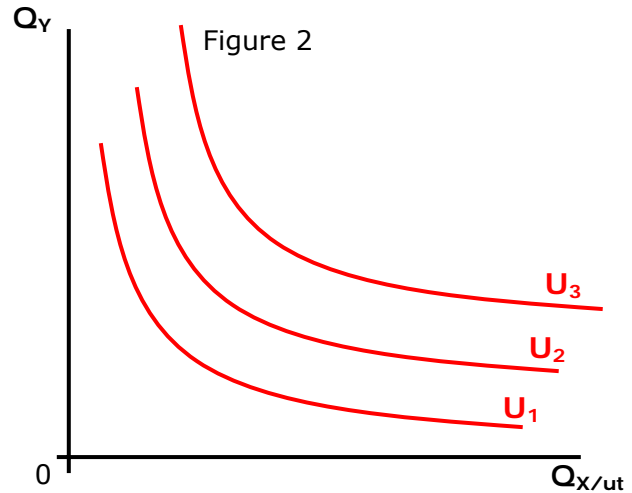
1. In Figure 1 the budget constraint can be constructed given M , P_x and P_y . Remember that an increase in income will shift the budget constraint to the right (or out) while a decrease in income will shift it to the left (or in) without changing its slope. A change in P_x or P_y (with no change in M) will rotate the constraint. A decrease in P_x (with no change in P_y or M) will move the Q_x intercept. A change in P_y , *ceteris paribus*, will change the Q_y intercept.



An increase in P_X will shift the Q_X intercept in toward the origin (an increase in P_Y will shift the Q_Y intercept in toward the origin). A decrease in P_X will shift the Q_X intercept away from the origin.

- The indifference curve is determined by the individual's preferences. Remember the goods may be complements, substitutes, have no utility, negative utility or positive utility. The shape of the indifference curve reflects the preferences of the individual. In practice the preferences cannot be measured directly but different characteristics of the buyer may be correlated with their preferences. Age, gender, religion, ethnicity, etc may provide information about the preferences of an individual.

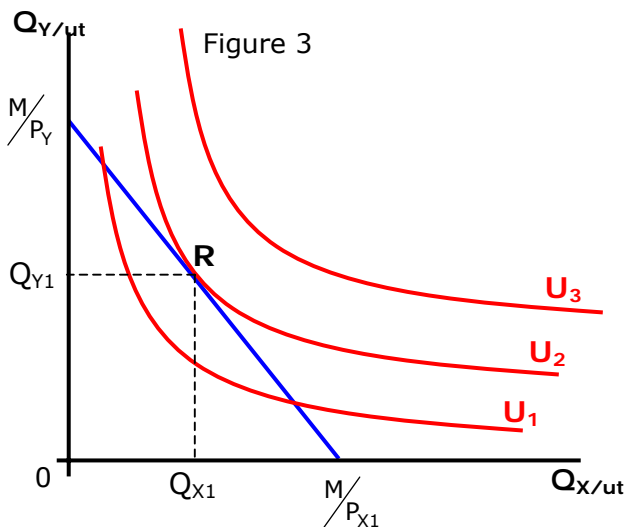
The indifference curves for an individual are shown in Figure 2. Remember that the shape of the indifference curves reflect the individual's



preferences, that there are an infinite number of them, that higher (further from the origin) indifference curves are preferred to lower ones (closer to the origin).

3. Consumer equilibrium and Utility maximization

The objective of the individual is to purchase a bundle of goods (Q_X and Q_Y) that is one the budget constraint and the highest possible indifference curve. The conditions that insure the maximization of utility given M, P_X and P_Y is the slope of the indifference curve is equal to the slope of the budget constraint:



$$MRS_{XY} = \frac{P_X}{P_Y}$$

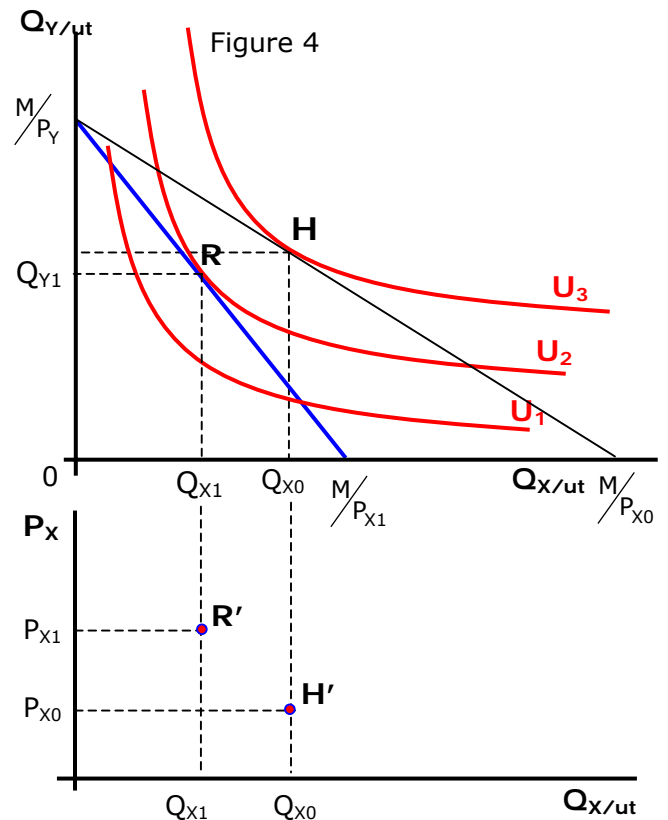
The optimal combination of good X and Y given income, prices and preferences is shown in Figure 3. At point R the budget constraint is tangent to the indifference curve U_2 . Given income (M) and the set of prices (P_{X1}, P_Y), U_2 is the highest indifference curve possible. Bundles that result in U_1 utility can be purchased but U_1 is not the maximum level of satisfaction attainable. U_3 is a higher level of satisfaction but cannot be reached given the budget constraint.

The bundle at point R (Q_{X1} and Q_{Y1}) is an equilibrium condition. Given income, prices and preferences the individual buyer has no incentive to alter their behavior. At a price of P_{X1} the consumer will purchase Q_{X1} , *ceteris paribus*. The price-quantity relationship ($P_{X1}-Q_{X1}$) represents one

point on the demand curve or demand schedule for good X ($Q_x = f(P_x)$, *ceteris paribus*).

4. To find other points on the demand curve or schedule for good X, change the price of X (P_x) and leave income (M), the price of good Y (P_y) and preferences (the indifference curves) unchanged.

5. In Figure 4 the price of good X is decreased to P_{x0} . This rotates the budget constraint out. The individual reacts by changing their purchases of good X (and in this case also good Y. This change in the amount of good Y purchased when the P_y is unchanged represents a change in the demand or shift in the demand of good Y and is significant.) Due to the decrease in P_x , a larger quantity Q_{x0} is purchased.



The bundle of goods represented at point R identifies a price-quantity (P_{x1} , Q_{x1}) combination that can be plotted as a point (R') on the demand curve in the lower panel of Figure 4. The lower price of X (P_{x0}) results in a new budget constraint. The individual reacts to the new circumstances (lower price of X, *ceteris paribus*) and moves to a new equilibrium at point H in the top panel of Figure 4. Point H represents a new price-quantity combination, P_{x0} , Q_{x0} (P_y , M and preferences are unchanged). The new price-quantity relation (P_{x0} , Q_{x0}) can be graphed on the lower panel at point H' . Two points on the demand for good X have been determined. Continue changing P_x (hold P_y , M and preferences constant) to locate other price-quantity combinations on the demand curve for good X.

6. Demand for good X

In Figure 5 the demand for good X is shown where $Q_x = f(P_x)$, *ceteris paribus*. There are two interpretations of the demand function.

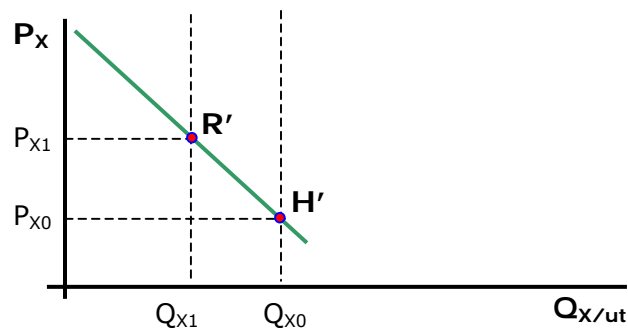


Figure 5

- a) Demand may be regarded as a "schedule of quantities that will be purchased at a schedule of prices in a given interval of time, *ceteris paribus*. This definition may be stated as $Q_x = f(P_x)$, *ceteris paribus*.
- b) Demand can also be perceived as the "maximum prices the buyer is willing and able to pay for each unit of the good, *ceteris paribus*. This form of the demand curve can be stated:
- $$P_x = f(Q_x), \text{ ceteris paribus}$$

Problems:

Given the demand curve $Q = 48 - 2P$,

- 1) State demand where P is a function of Q.
- 2) What is the Q-intercept?
- 3) What is the P-intercept?
- 4) Graph the demand function.
- 5) What is the slope of the demand function?

(This is a trick question, you have to specify whether it is $Q=f(P)$ or $P=f(Q)$)

II. The price consumption curve (PCC)

The PCC provides the same information that is in the demand curve; it is just in a different form. The PCC is constructed in much the same way as demand curves; the price of the good in question (P_x) is changed while P_y , M and preferences are unchanged. In Figure 6 a PCC is shown.

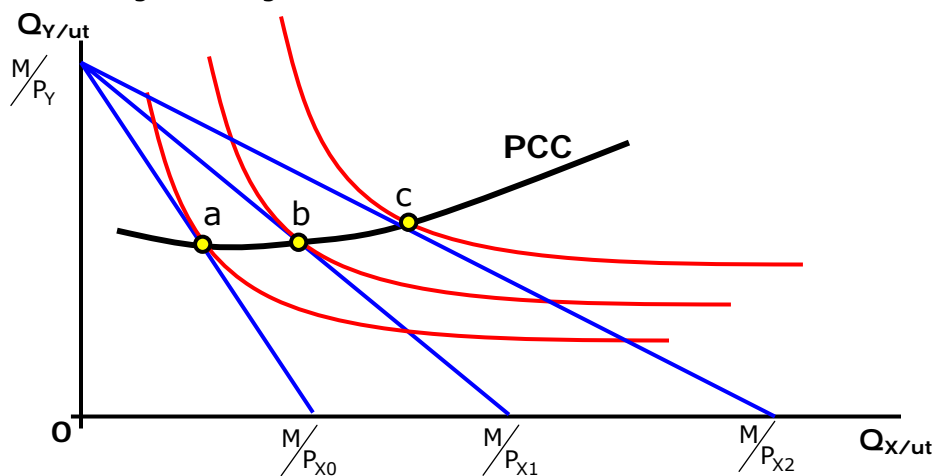


Figure 6

The PCC is the locus of equilibrium bundles of goods that results when the price of one good is changed and all other things are unchanged (M and preferences). As the price decreases from P_{x0} to P_{x1} to P_{x2} the individual adjusts the bundles they purchase. The $MRS = \frac{P_{xi}}{P_y}$ at each point on the PCC. When M and P_y remain constant, for the prices P_{x0} , P_{x1} , and P_{x2} the utility optimizing bundles will be at points a , b and c respectively. Later we will establish that when the PCC is negatively sloped the price elasticity of demand will be elastic. Where the PCC has a slope of 0, the price elasticity of demand is unitary. Inelastic demand will be reflected in a positively sloped PCC. The demand curve can be derived directly from the PCC.

III. The Income Consumption Curve (ICC)

The ICC is the locus of equilibrium bundles of goods when the income (M) changes and the prices and preferences remain fixed. It is a way to measure the

responsiveness of buyers to changes in income. The ICC provides the same information about the relationship between demand and income as the Engel curve.

A. A normal good

The ICC is constructed using the indifference curves and changes in income. The ICC for a "normal" good is constructed in Figure 7.

In Figure 7 the income is increased from M_1 to M_2 to M_3 with no changes in the P_X or P_Y . As the income increases the budget constraint shifts out with no change in its slope (the relative prices were unchanged). Given the increases in income the individual reacted by altering their purchases. While the prices of X and Y remain constant, the utility optimizing bundles for income levels M_1 , M_2 , and M_3 will be e, h and f respectively. In the case of a normal good (actually both good X and Y are normal goods), the ICC will have a positive slope. An increase in income results in an increase in the quantity purchased.

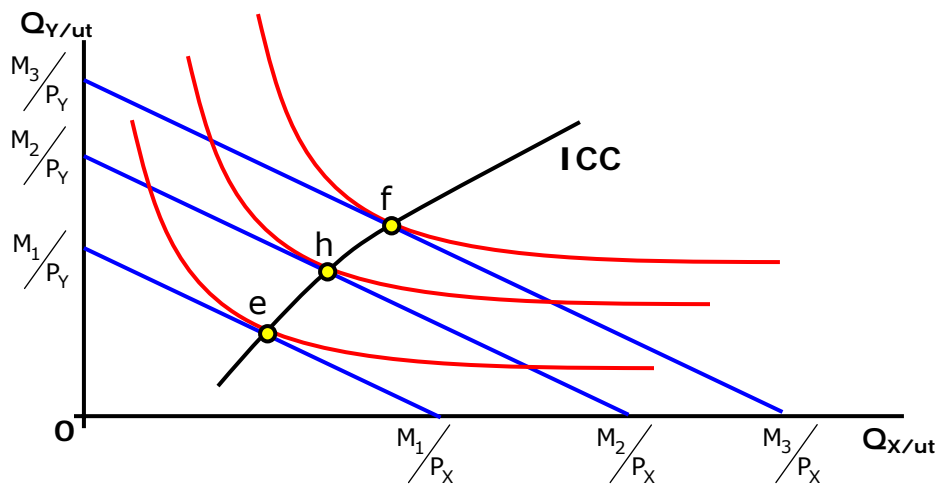


Figure 7

B. An **Inferior Good** is shown in Figure 8. As the income increases from M_1 to M_2 to M_3 , the utility optimizing bundles are λ , α and β . Note that as income increases the quantity of good X purchased decreases. The ICC will have a negative slope.

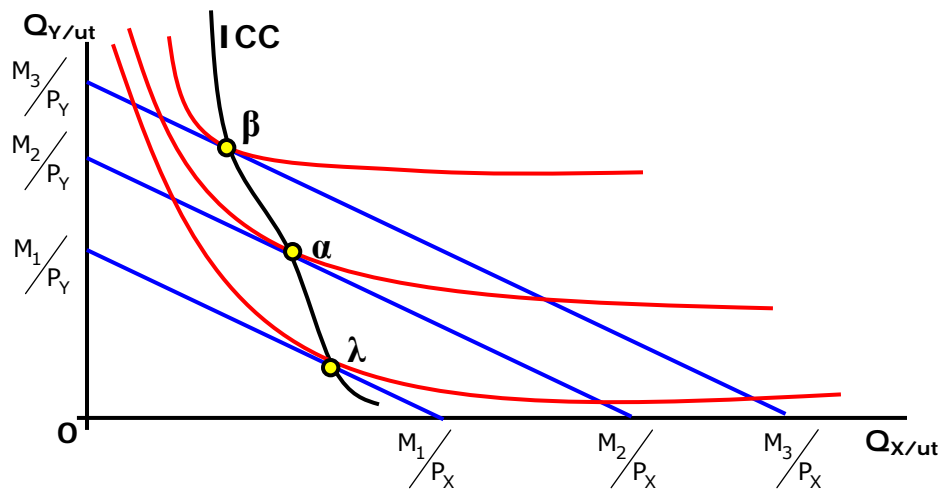


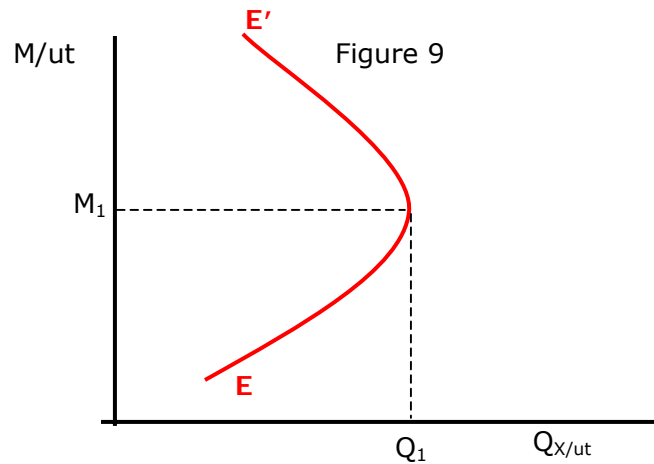
Figure 8

C. A **superior good** is one where the percentage increase (decrease) in the expenditure on a good is greater than the percentage increase (decrease) in income. These goods are often referred to as "luxury goods." In Figure 8 (above) good Y is a superior good. When constructing indifference curves on an indifference map, if one good is a superior good the other must be an inferior good.

D. **Engel Curve**

When the prices of the goods (P_X and P_Y) remain constant and income (M) changes, the quantity of a good purchased can be stated as a function of M : $Q = f(M)$ *ceteris paribus*. This is analogous to the demand curve. The Engel curve (sometimes referred to as "income demand") can be constructed from the ICC. If the good is a normal or superior good it will have a positive slope. An inferior good has a negatively sloped Engel curve. It is possible that the Engel curve may be normal in one income range and inferior at higher levels of income. Figure 9 shows an Engel curve for a good such as hamburger meat.

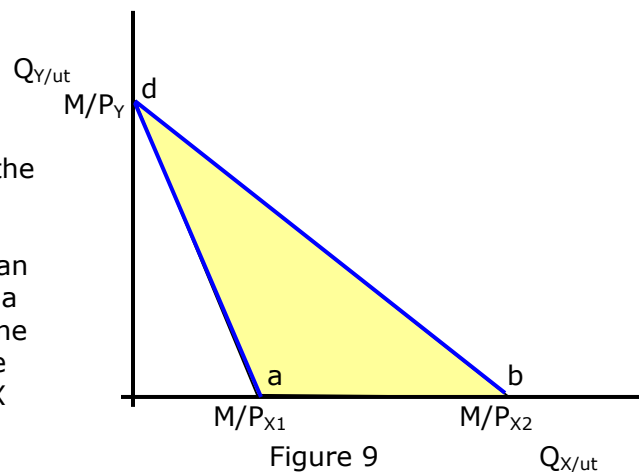
At incomes less than M_1 the good is a normal good (individuals buy more as income increases). At income levels above M_1 the good becomes an inferior good.



IV. Income and Substitution Effects and Downward Sloping Demand Schedules

The demand schedule can be constructed directly from the PCC. Generally, demand schedules are negatively sloped, i.e. there is an inverse relationship between price and quantity. This inverse relationship is due to two factors.

A. The **income effect** is the change in the quantity purchased which is the result of a change in "real" income that occurs when the price of a good changes and nominal income (M) is held constant. A decrease (increase) in the price of a good when M is held constant will increase (decrease) the real income. This can be shown by the area between two budget constraints, given a change in P_X , *ceteris paribus*. In Figure 9 the area abc represents additional bundles of goods X and Y that can be purchased due to an increase in real income caused by a decrease in the price of good X from P_{X1} to P_{X2} .



B. The **substitution effect** is the tendency of an individual buyer to substitute relatively less expensive goods for relatively more expensive goods. The MRS is one of the determinants of the substitution effects. For normally shaped indifference curves the substitution effect

C. Measuring income and substitution effects for normal goods graphically
 The income and substitution effects can be shown with budget constraints and an indifference map.

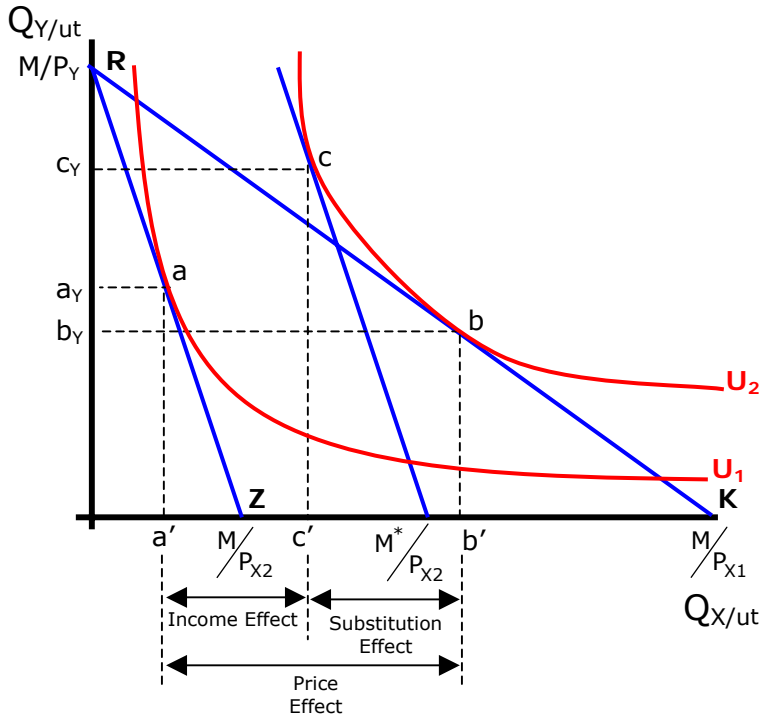


Figure 10

In Figure 10 the consumer begins with an income of M and the prices of Y and X at P_Y and P_X respectively. The initial budget constraint is line RK . The consumer maximizes utility at point b , purchasing b'_x of good X and b'_y of good Y

$$\left(MRS = \frac{P_X}{P_Y} \right)$$

An increase in the price of good X to P_{X2} rotates the budget constraint in to line RZ . The consumer alters their purchases. The optimal bundle is a'_x of good X and a'_y of good Y at point a .

The decrease in quantity of good X from b'_x to a'_x represents the "price effect."

The increase in P_X reduces the real income (M is constant). A hypothetical increase in real income to move the consumer back to the original indifference curve (U_2) will shift the budget constraint out to a new Q_X intercept M^*/P_{X2} . Given the new relative prices of good X and Y with a comparable real income M^* , the consumer will be in equilibrium at point c .

Price effect is the quantity $b'_x - a'_x$
 Income effect is the quantity $c'_x - a'_x$ (the change in quantity caused by a change in real income when ΔP_X)
 Substitution effect is the quantity $b'_x - c'_x$

1. The price effect or total effect is the change in the quantity of good X purchased caused by a change in the price of good X .
2. The income effect is that part of the price effect or change in the quantity of X purchased that is attributable to the change in real income that occurred when the price of good X changed. In Figure 10 note that the income effect lies between the "new" budget constraint and the hypothetical budget constraint (with the new price and M^*).
3. The substitution effect is that part of the price effect or change in the quantity of good X purchase that is due to the substitution of good Y for good X . In Figure 10 note that the substitution effect is measured between two points one the original indifference curve, it is the MRS.
4. In Figure 10 both good X and Y are normal goods.

Problem:

In Figure 11, income is M , P_X and P_Y are given. Show a new budget constraint for a decrease in the price of good X to P_{X0} . Then construct a new indifference curve to show the price effect of the decrease in the price of good X to P_{X0} . Draw in the hypothetical budget constraint that will put the consumer back on their original indifference curve. Show the income and substitution effects. How is this different from a price increase?

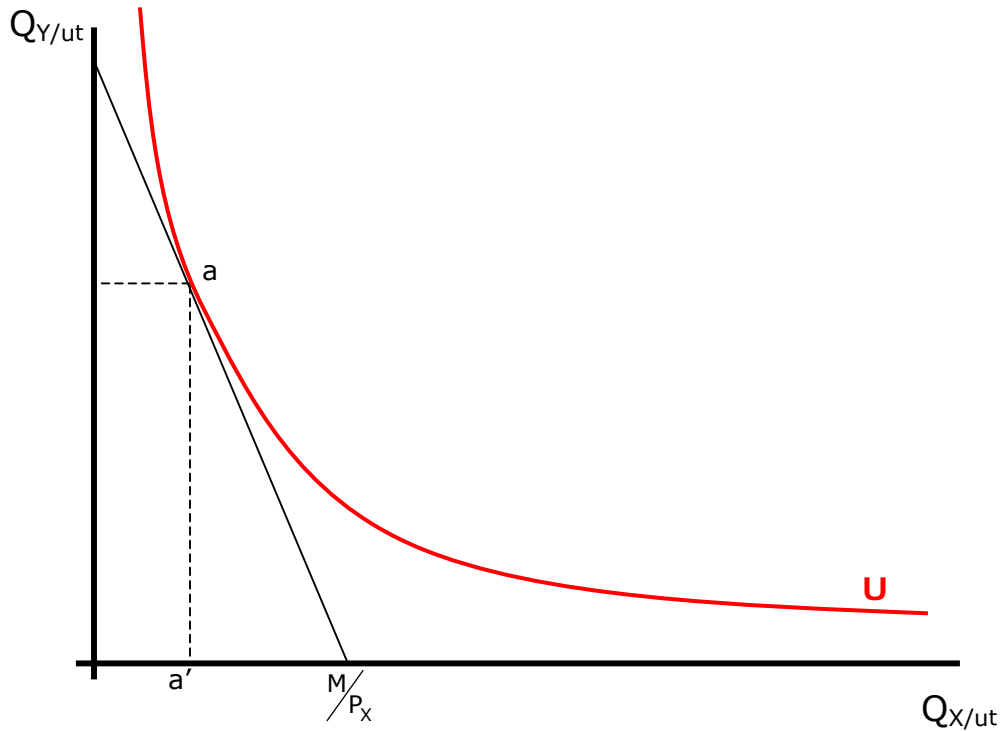


Figure 11

Does your graph look something like the graph in Figure 12?

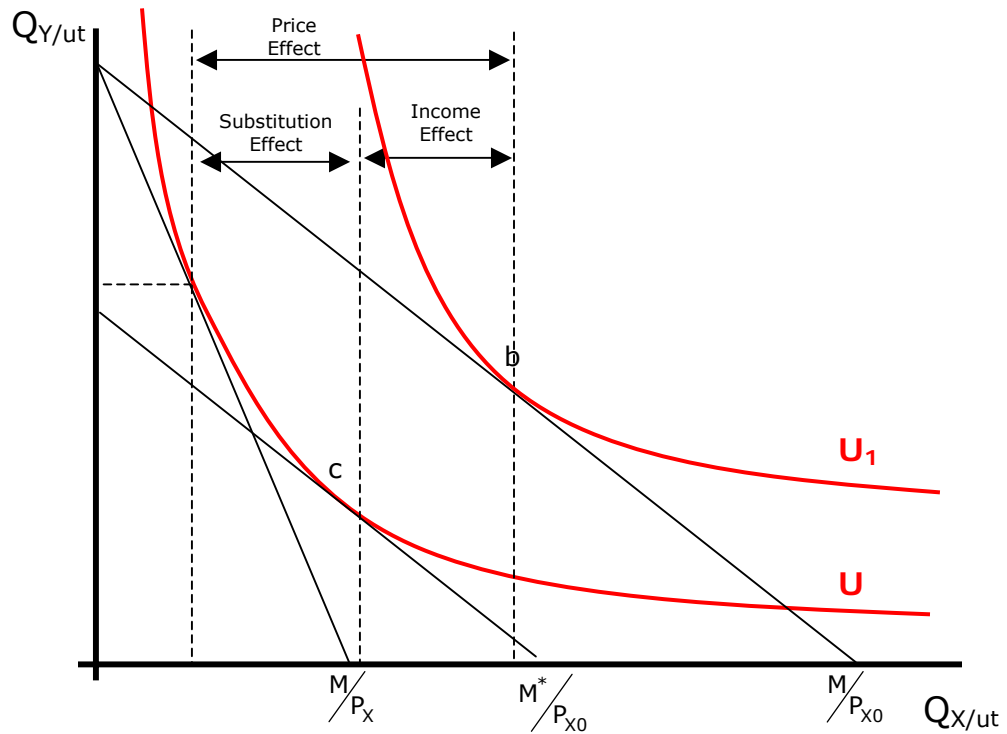


Figure 12

Notice that the income and substitution effects are on different sides when there is a price decrease rather than a price increase.

D. Income and Substitution Effects for an Inferior Good

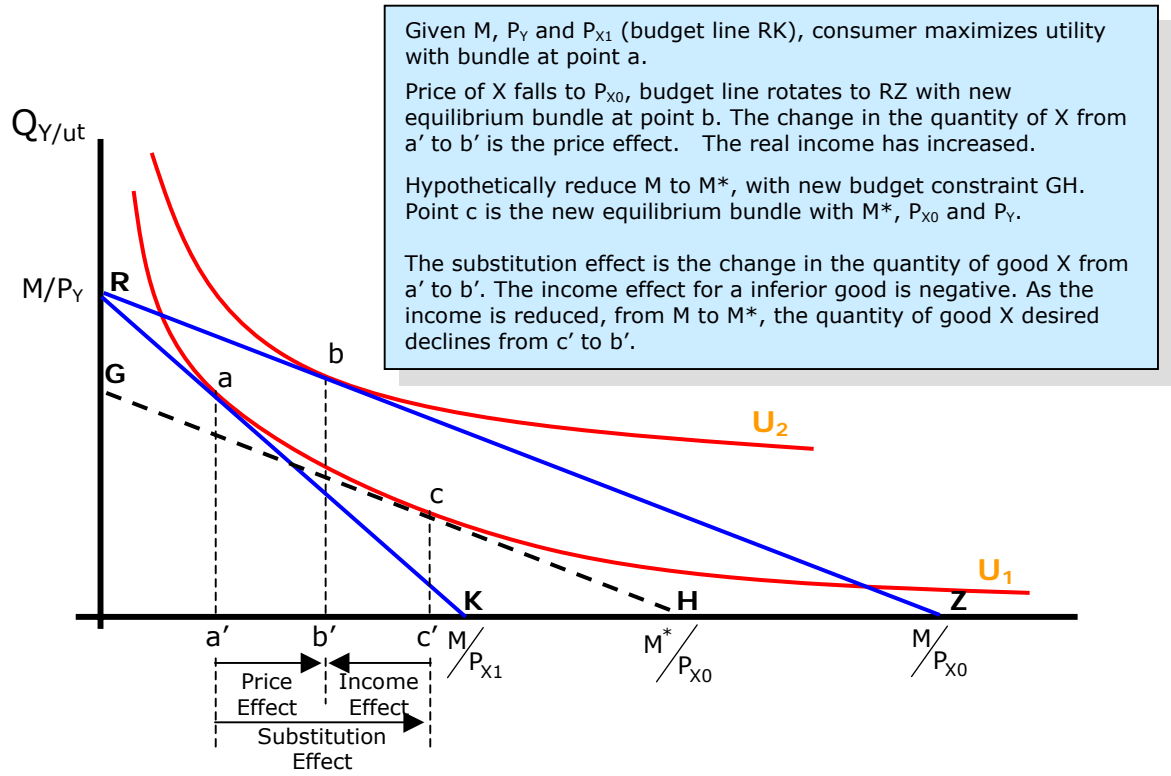


Figure 13

V. Individual and Market Demand

- A. In a setting where property rights are "exclusive," i.e. if an individual gets a unit of good X (like a loaf of bread), all the benefits and costs associated with the production and consumption of that good (loaf of bread) are exclusive to the individual. (Collective goods and goods with externalities present a problem.)
- B. The market demand is the "horizontal summation" of the individual demand schedules of all individuals in the market.

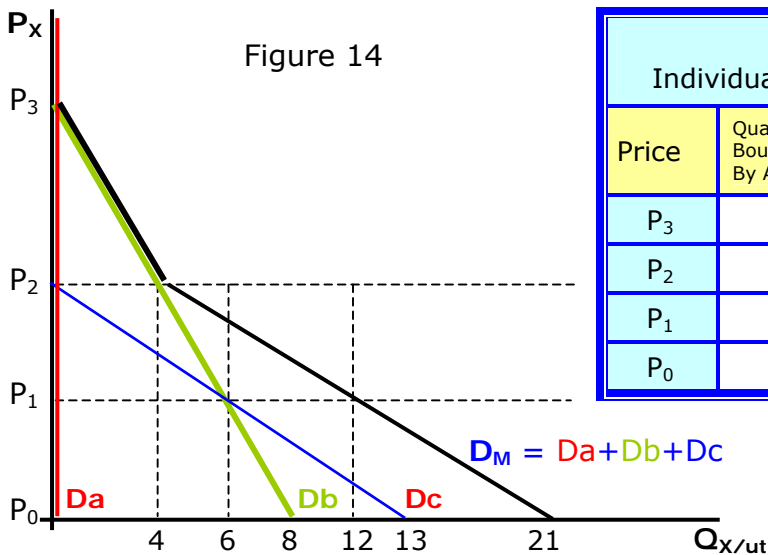


Figure 14

Price	Quan Bought By Ann	Quan Bought By Bob	Quan Bought By Cab	ΣD_i Market demand
P_3	0	0	0	0
P_2	0	4	0	4
P_1	0	6	6	12
P_0	0	8	11	21

In Figure 14, there are 3 potential buyers, Ann (a), Bob (b) and Cab (c). Each may have different preferences or incomes that cause them to react differently to prices by buying different quantities. Ann will not buy any of good X at any price. Bob will not buy at a price of P_3 or above. At a price of P_2 Bob will buy 4 units. If the price were P_1 , Bob would buy 6 units. Each persons demand schedule is shown in Figure 14 and in accompanying Table 14. The market demand is the horizontal summation of the individual's demands. If Bob wants 6 units at P_1 and Cab also wants 6 units at that price, 12 units will be demanded in the market.

VI. The Demand Model

The the nature of a market demand function is influenced by 5 things

- The income of the buyer or buyers (M)
- Price of all related goods (P_Y ; complements, substitutes and numeraire goods)
- The price of the good being considered (P_X)
- Preferences of the buyer
- The number of buyers

The demand function can be expressed;

$$Q_x = f(P_x, P_y, M, \text{preferences, number of buyers, } \dots)$$

When $P_y, M, \text{preferences, number of buyers, } \dots$ are held constant, we are left with the demand schedule or demand curve:

$$Q_x = f(P_x) \text{ ceteris paribus}$$

A. Dependent variable (Q_x)

1. $Q_{x/ut}$ represents the amount of good X that will be purchased during a period of time (ut) given a set of values for $P_x, P_y, M, \text{ preferences, number of buyers and other things } (\dots)$.
2. To simplify, the quantity of X is usually defined as a continuous variable.
3. Goods may be measured in a variety of ways. For many good out put is in units, in other cases it may be ton/miles, passenger/miles, trips, tickets, etc.
4. Units, tens, hundreds, thousands, etc. may be used.

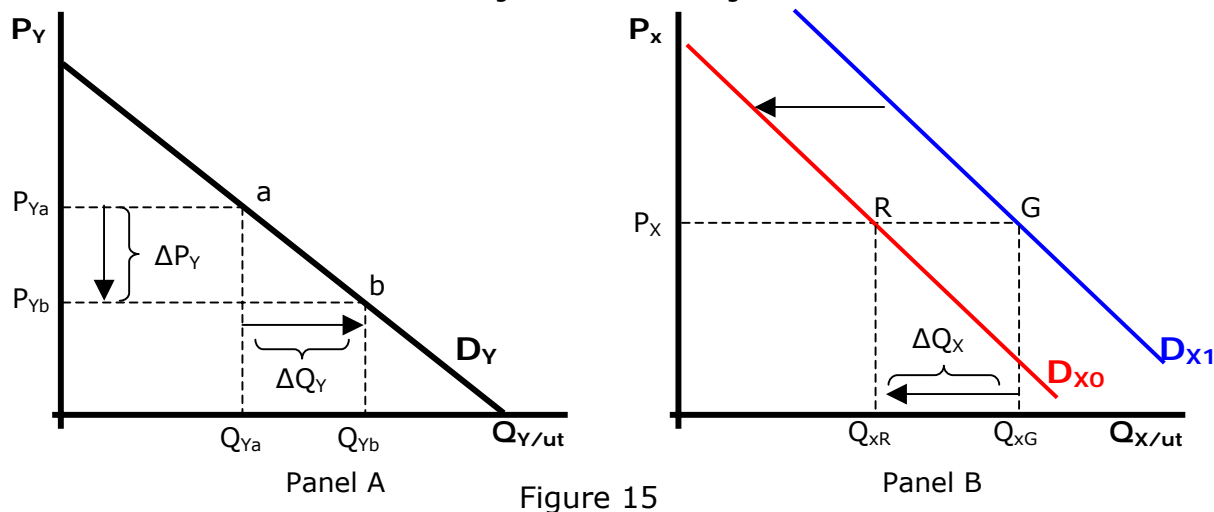
B. Independent variable, price of X (P_x)

1. In most economic models it is assumed that there is no price discrimination. All units in each time period are sold at the same price.
2. Prices are often treated as continuous variables
3. a change in the price of the good P_x results in a "change in quantity demanded" or a movement along a demand curve or schedule.
4. The responsiveness of buyers to changes in the price of the good is measured by "price elasticity of demand."

C. Independent variable, prices of related goods (P_y)

1. A change in the price of a good that is a substitute for good X will "shift" the demand schedule or curve. This is a "change in demand."
 - a) An increase in the price of Y will result in an "increase in the demand of good X," it will shift the demand schedule of good X to the right. More of good X will be desired at each price of X (P_x).
 - b) A decrease in the price of Y will result in a "decrease in the demand for good X," it will shift the demand schedule of good X to the

left. Buyers are willing and able to purchase smaller quantities of good X at each price of good X.
 An example of the effect of a change in the price of a substitute good is shown in Figure 15.



In Figure 15 the demand schedules for goods X and Y are shown. In panel A, a decrease in the price of good Y will cause an increase in the quantity demanded of good Y from Q_{Ya} to Q_{Yb} .
 As the buyer purchases more of good Y at the lower price of Y, the amount of good X they are willing to purchase at each price of X will decrease, "shift to the left" from D_{X1} to D_{X0} .
 The decrease in the price of good Y decreased the demand for good X.
 (Show on a graph what an increase in the price of good X will do to the demand for good Y.)

2. Changes in the price of a **complementary good** will shift or change the demand for good X.
 - a) An increase in the price of a complement will decrease the demand for good X.
 - b) A decrease in the price of a complement will increase the demand for good X.
 - c) Draw the graph that shows the effects of a change in the price of a complementary good.
3. A change in the price of any good that a consumer buys will shift the budget constraint and will probably shift the demand for good X.
4. The relationships between goods can be measured using the **"cross elasticity of demand."**
 - a) $E_{XY} > 0$, \Rightarrow a substitute
 - b) $E_{XY} < 0$, \Rightarrow a complement
5. Remember that things like interest rates represent a "price" for borrowed funds. A change in interest rates (the price of a complementary good) can shift the demand for houses or cars.

D. Income as an independent variable **(M)**

A change in nominal income will shift the budget constraint and alter the purchases of the individual. This was shown with the ICC and the Engel curve. Goods tend to be placed into one of four categories with respect to the relationship between income and the demand for a good.

a) **Inferior Goods**

When an increase in income reduces the demand for a good it is regarded as an inferior. A decrease in income will increase the demand for an inferior good. There is an inverse relationship between income and the demand for the good. Beans, *Top Ramen* noodles, peanut butter are examples of inferior goods for students.

b) **Normal Goods**

There is a positive relationship between income and the demand for a normal good. An increase (decrease) in income results in an increase (decrease) in the demand for a normal good.

c) **Superior Goods**

A superior good is a special case of a normal good. The percentage change in the demand for the good is greater than the percentage change in income.

E. **Preferences** as an independent variable

It is not possible to measure and describe preferences directly. Preferences can be deduced and correlated with other variables. Age, gender, religion, ethnicity, seasonal variations, time, and other variable may be correlated to the preferences for some goods. When estimating demand function it may be useful to use proxies that are correlated with preferences. Expectations about the future may be included under preferences.

F. The **number of potential buyers** or buyers in the market

The number of buyers in a market may be the population of a particular geographic area or members of a specific group (age, religion, ...)

G. **...**, Theory requires the abstraction from reality. A model does not include all the elements of a set of phenomena; it only includes the "most" important or relevant aspects. The three dots are used to indicate there may be other variables that are not included in the model.

VII. Estimation of Demand

While it is not possible to "prove" causation, it is possible to statistically show correlations between variables. An empirical estimate of the demand function can be made in principle. In practice, it is plagued by many problems with data and what is called the "identification problem."

A. Empirical data may not always be what it seems. The collection of data may be expensive and is not always reliable. Generally, the data may be collected as "cross-sectional" or "time-series" data.

1. Cross-sectional data is collected among several populations (geographic regions, or different classes or groups) at the same time. There may be differences between the different classes that you must control for.
2. Time-series data is collected from the same population over time. Each time period represents an observation.
3. It is necessary to have sufficient observations to estimate the correlations with some confidence.

B. Any interpretation of data or facts requires recognition of a pattern. If the data are truly random there is no pattern. The interpretation of data presumes there is an underlying structure that generated the data. Meaningful interpretation of a set of observations is made more difficult if there are other structures that could have generated the same data. In demand estimation the question is whether the observed facts were generated by changes in the demand equation, the supply equation or

changes in both supply and demand. There are a variety of statistical methods (two and three stage least squares as example) that may be used to resolve the identification issues.

In Figure 16 there are a series of price-quantity combinations labeled a-h. Each observation represents a price and a quantity that occurred in a given year.

An ordinary least squares method would suggest that demand as a price-quantity relationship could be shown by the line AB.

The observations may have been generated by shifting supply and demand functions over time. Observation "a" was due to supply S_1 and demand D_1 . Observation "g" was the result of an increase in both supply and demand to S_2 and D_2 .

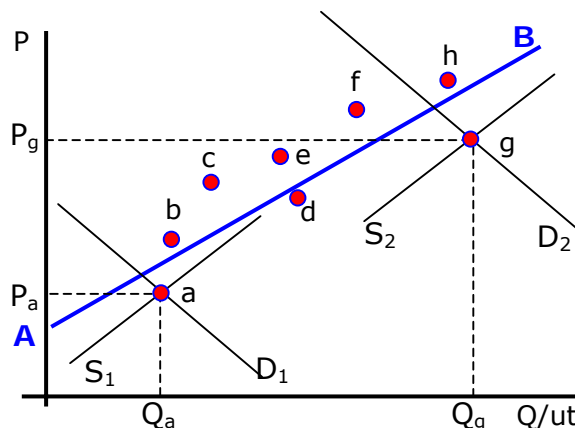


Figure 16

c. The demand equation might be estimated:

$$Q_x = f(P_x, P_y, P_z, M, \text{pop}, \epsilon)$$

Where:

Q_x = the quantity of good X (dependent variable, may be in units, tons, etc.)

P_x = the price of good X (independent variable, may be the average price of good X per unit)

P_y = the price of a related good Y (independent variable, may be the average price of good Y per unit)

P_z = the price of a related good Z

M = income (independent variable, may be average disposable income)

pop = the population of the market

ϵ = an error term

The estimates of the coefficients on the independent variables are estimated using statistical methods:

$$Q_x = 20 - 2P_x + 4P_y - 1.5P_z + .1M + .005\text{pop} + \epsilon$$

$$P_y = \$10 \text{ (average price of good y)}$$

$$P_z = \$8 \text{ (average price of good Z)}$$

$$M = \$1000 \text{ (average weekly disposable income)}$$

$$\text{pop} = 10,000 \text{ people}$$

The demand schedule or curve can be expressed:

$$Q_x = 198 - 2P_x, \text{ ceteris paribus}$$

The demand schedule was calculated by substituting the values for each of the independent variables into the demand function or model.

Notice that the Q-intercept contains the error term and all the effect of all the independent variables except P_x . If any of the variable other than P_x change the demand Q-intercept of the demand schedule will change. This is a change in demand. The demand schedule is shown in Figure 17.

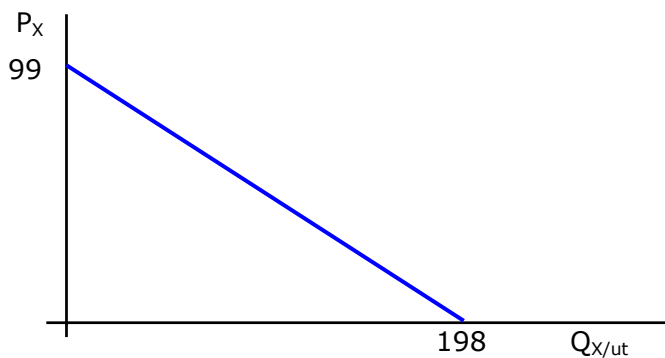


Figure 17

Table 17 Demand for Good X	
P_x	Q_x
\$99	0
\$75	48
\$50	98
\$25	148
\$10	178
\$0	198

The coefficient on each of the variables describes how a change in the variable will alter the demand schedule for good X.

- Good Y is a substitute for good X. An increase (decrease) in the price of good Y (P_Y) will increase (decrease) the Q_X , because the sign on the coefficient for P_Y is positive. For a \$1 change in the price of good Y, the quantity of good X will change by 4 units in the same direction.
- Good Z is a complement to good X. The sign on the coefficient on P_Z is negative. A \$1 increase (decrease) in the price of good Z will decrease (increase) the quantity of good X purchased by 1.5 units.
- Since the sign on the coefficient of the income variable (M) is positive, good X is a normal or superior good. An increase (decrease) in M will increase (decrease) Q_X . An inferior good will have a negative coefficient on the M variable.
- The coefficient on the pop variable suggests that for every 200 people you will sell 1 unit of good X.

Problems:

- Using the demand model: $Q_x = 20 - 2P_x + 4P_y - 1.5P_z + .1M + .005pop + \epsilon$
 - Graph the demand schedule when the P_z increases to \$50.
 - If after the P_z went to \$50, the income (M) decreased to \$800, what will the new demand schedule for good X be?

VIII. Elasticity

Elasticity is a measure of how responsive a dependent variable is to changes in an independent variable. Elasticity is defined as a ratio of the percentage change in a dependent variable "caused" or associated with a percentage change in an independent variable:

$$\text{Elasticity} = \epsilon = \frac{\% \Delta \text{ in a dependent variable}}{\% \Delta \text{ in an independent variable}}$$

A. "Own" Price Elasticity of Demand

The price elasticity of demand (ϵ_p) is a measure of how responsive buyers are to changes in the price of a good:

$$1. \quad \epsilon_p \equiv \frac{\% \Delta Q_x}{\% \Delta P_x}$$

a) Point Price elasticity

$$\epsilon_{P(\text{point})} = \frac{\frac{\Delta Q}{Q_1}}{\frac{\Delta P}{P_1}} = \frac{\Delta Q}{\Delta P} \left(\frac{P_1}{Q_1} \right), \text{ or } \epsilon_P = \frac{\partial Q_x}{\partial P_x} \left(\frac{Q_x}{P_x} \right)$$

b) Arc or Average Price Elasticity of Demand

$$\epsilon_{P(\text{arc})} = \frac{\frac{\frac{\Delta Q}{Q_1 + Q_2}}{2}}{\frac{\frac{\Delta P}{P_1 + P_2}}{2}} = \frac{\Delta Q}{\Delta P} \left(\frac{P_1 + P_2}{Q_1 + Q_2} \right), \text{ or } \frac{\partial Q_x}{\partial P_x} \left(\frac{\Sigma P_x}{\Sigma Q_x} \right)$$

c) Price elasticity measures the responsiveness of buyers to changes in the good's "own" price. It is a measure of a movement along a demand function and is determined by:

(1) The slope of the demand function, $\frac{\Delta Q}{\Delta P}$, (when $Q = f(P)$)

(2) The location on the demand function, P (or average P) and Q (or average Q).

Problems:

- 1) Given $Q_x = 20 - 2P_x + 4P_y - 1.5P_z + .1M + .005\text{pop}$,
 - $P_y = \$10$ (average price of good y)
 - $P_z = \$8$ (average price of good Z)
 - $M = \$1000$ (average weekly disposable income)
 - pop = 10,000 people
 - a. Calculate the point price elasticity of demand when $P_x = \$25$.
 - b. Calculate the price elasticity when $P_x = \$75$.
 - c. What is point price elasticity of demand when $P_x = \$49.50$?
- 2) Given the same demand function,
 - a. calculate the average or arc elasticity between the prices of \$20 and \$30.
 - b. Between the prices of \$80 and \$50.

2. Interpretation of ϵ_P

a) The value of ϵ_P will be negative (so long as the demand is negatively sloped)

b) When $|\epsilon_P| > 1$, the demand is "elastic"

$$\text{The } |\% \Delta Q_x| > |\% \Delta P_x|$$

Price (P) and total revenue (TR) will move in "opposite directions" (an increase (decrease) in P will reduce (increase) TR)

c) When $|\epsilon_P| < 1$, the demand is "inelastic"

$$\text{The } |\% \Delta Q_x| < |\% \Delta P_x|$$

Price (P) and total revenue (TR) will move in the "same direction" (an increase (decrease) in price will increase (decrease) TR)

d) When $|\epsilon_P| = 1$, the demand is "unitary"

$$\text{The } |\% \Delta Q_x| = |\% \Delta P_x|$$

When $|\epsilon_P| = 1$, TR will be a maximum

3. Quick and Easy – “the mid-point”

For any linear demand curve, the “mid-point” will be unitarily elastic ($|\epsilon_p| = 1$) and TR will be a maximum.

Figure 18

In the lower half of Figure 17, the demand for good X is shown as a linear function. Q_x is a function of P_x , however the graph is drawn with P_x on the vertical axis and Q_x on the horizontal.

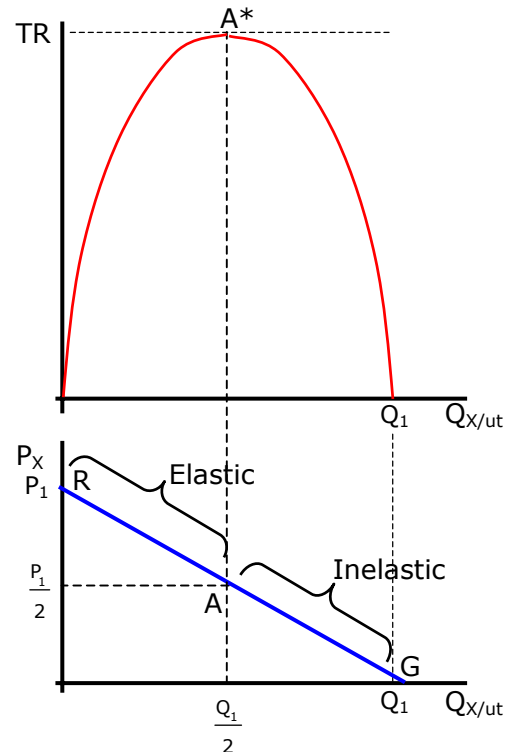
The linear demand schedule, RG has a quantity-intercept at Q_1 and a price-intercept at P_1 .

Divide the demand schedule in half by dividing Q_1 by 2 or P_1 by 2. Point A is the mid point of the demand schedule.

At point A the price elasticity of demand will be -1 or unitarily elastic. Note that in the top panel the TR function ($TR=f(PQ)$) is a maximum at A^* .

In the price range from $P=0$ to $P=\frac{1}{2} P_1$, the demand function will be inelastic ($|\epsilon_p| > 1$). Note that both TR and the demand schedule have a negative slope while the Demand has a negative slope; P and TR are positively related ($+\Delta P \Rightarrow +\Delta TR$, $-\Delta P \Rightarrow -\Delta TR$).

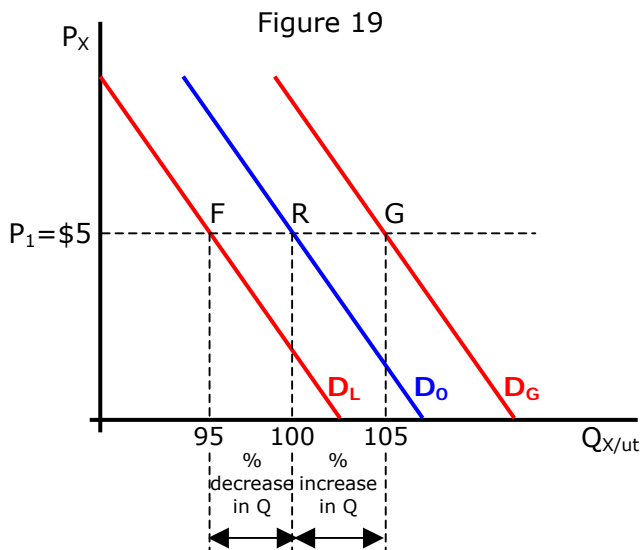
In the price range from $P=\frac{1}{2} P_1$ to P_1 , the demand will be elastic ($|\epsilon_p| > 1$). Price and TR are inversely related. TR is positively sloped while demand is negatively sloped. P and TR move in opposite directions.



B. Income Elasticity

Income elasticity is a measure of how a change in nominal income will shift the demand for a good.

$$\epsilon_M \equiv \frac{\% \Delta Q_x}{\% \Delta M} = \frac{\Delta Q}{\Delta M} \left(\frac{M}{Q} \right), \text{ or } \frac{\partial Q_x}{\partial M_x} \left(\frac{\Sigma M}{\Sigma Q_x} \right)$$



In Figure 19, D_0 represents the original demand for good X. At a price of \$5 the individual will purchase 100 units of Q_x .

If the income increases from \$500 to \$600 there is a 20% increase in income.

If the demand shifts to D_G (as income increases to \$600) there will be a 5% change in the quantity purchased. ϵ_M will be +.25. In this case, the good is a **normal good**.

If the demand shifted to D_L as a result of the \$100 increase in income (a 20% increase to \$600), the $\% \Delta Q = -5\%$. Income elasticity will be $\epsilon_M = -.25$. In this case it is an **inferior good**.

- If there is a negative relationship between a change in income and the quantity of the good, it is called an "inferior good." Both the ICC (income consumption curve) and Engel Curve will be negatively sloped.

$$\epsilon_M < 0, \text{ inferior good}$$

- A positive relationship between income and Q_X is:
 - A normal good if $-1 < \epsilon_M < 0$, the ICC and Engel Curve are positively sloped.
 - A "superior" good if $\epsilon_M > 1$

C. Cross Elasticity of Demand

Cross elasticity is used to identify and describe the relationship between goods. It is defined as:

$$\epsilon_{XY} \equiv \frac{\% \Delta \text{ in Quantity of good X}}{\% \Delta \text{ in Price of good Y}} = \frac{\Delta Q_X}{\Delta P_Y} \left(\frac{P_Y}{Q_X} \right), \text{ or } \frac{\partial Q_X}{\partial P_Y} \left(\frac{\Sigma P_Y}{\Sigma Q_X} \right)$$

1. Substitutes

Goods are substitutes when an increase (decrease) in the price of good Y causes an increase (decrease) in the amount of good X that is purchased at each price.

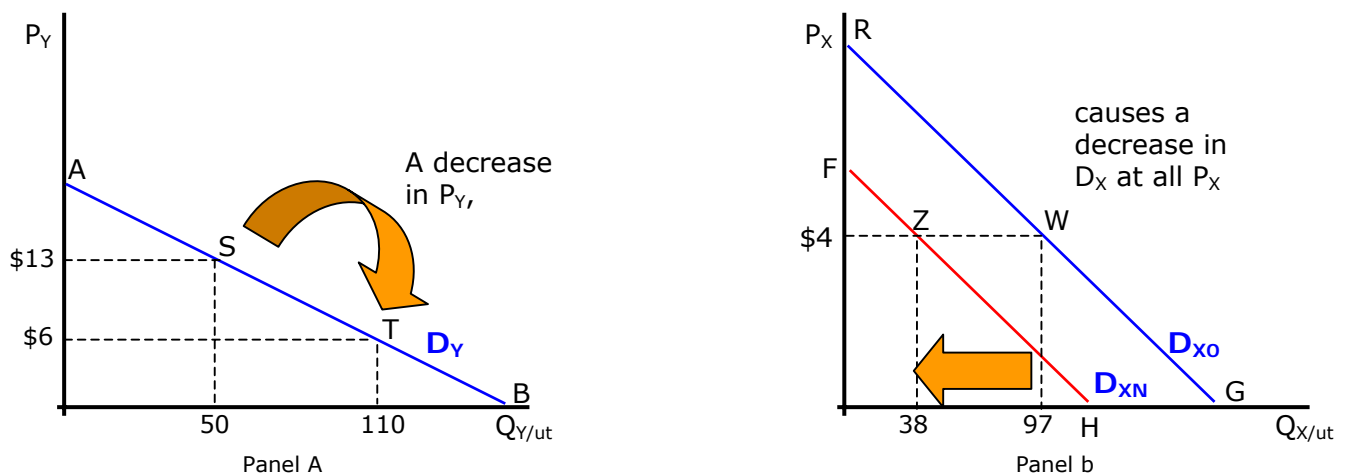


Figure 20

In Figure 20 there are graphs for two goods, Y and X. These goods are substitutes. Each has a demand schedule. In Panel A, the demand for good Y is shown as line AB and is labeled D_Y .

If the price of good Y is \$13 ($P_Y=13$), the original demand for good X will be D_{X0} and is shown in Panel B as line RG. The demand for good X is a function of P_X , P_Y , M, Preferences, #buyers, Remember that the demand curve of schedule for good X is $Q_X=f(P_X)$ *ceteris paribus*. A change in P_Y will result in a new demand curve for good X.

In Panel A, a decrease in P_Y from \$13 to \$6 will increase quantity demanded of good Y from 50 to 110 units. This is shown as a movement along the demand for Y from point S to point T.

As the buyers purchase more of good Y, they will want less of good X at each price. As the price of Y falls \Rightarrow buy more good Y \Rightarrow buy less of substitute good X at each price (in Panel B \$4 is example). This is shown as a decrease in demand for good X from D_{X0} to D_{XN} . At price of \$4 the quantity of good X purchased falls from 97 units to 38 units.

The decrease in the price of Y ($\Delta P_Y < 0$) resulted in an decrease in the quantity of X purchased ($\Delta Q_X < 0$). Since $\epsilon_{XY} =$ a ratio of $\% \Delta Q_X$ to $\% \Delta P_Y$, the $\epsilon_{XY} > 0$ (it is positive). When ϵ_{XY} is positive, this is evidence the goods are substitutes.

Problems:

- 1) Work out what happens when there is an increase in the price of good Y in Figure 20. Draw another graph to show the effects of the price increase. Will ϵ_{XY} be positive or negative?

(An increase in $P_Y \Rightarrow$ decrease in quantity demanded of Y \Rightarrow buy more X at each price, this is an increase (shift to right) in the demand for good X. Since ΔP_Y is positive and ΔQ_X is also positive, ϵ_{XY} will be positive.)

- 3) 2) Given the Demand for good X is:

$$Q_X = 20 - 2P_X + 4P_Y - 1.5P_Z + .1M + .005pop,$$

$P_Y = \$10$ (average price of good Y)

$P_Z = \$8$ (average price of good Z)

$M = \$1000$ (average weekly disposable income)

pop = 10,000 people

The P_X is currently $P_X = \$24$

- A) Calculate the ϵ_{XY}

By substituting in the values for P_Y, P_Z, M and pop, we have
 $Q_X = 198 - 2P_X$. When $P_X = 24, Q_X = 150$

$$\epsilon_p = \frac{\partial Q_X}{\partial P_Y} \left(\frac{P_Y}{Q_X} \right) = (+4) \left(\frac{10}{150} \right) = +.267$$

- B) Calculate ϵ_{XZ} .

By substituting in the values for P_Y, P_Z, M and pop, we have
 $Q_X = 198 - 2P_X$. When $P_X = 24, Q_X = 150$

$$\epsilon_p = \frac{\partial Q_X}{\partial P_Z} \left(\frac{P_Z}{Q_X} \right) = (-1.5) \left(\frac{8}{150} \right) = -.08$$

D. Applications of Elasticity

Frank reports some estimates of price, income and cross elasticities in the text.

Table A Price Elasticity Estimates of Short-run ϵ_p (Frank, p128)	
Good/Service	Price Elasticity
Green Peas	-2.8
Electricity	-1.2
Beer	-1.9
Movies	-.87
Air travel (foreign)	-.77
Shoes	-.70
Theatre, opera	-.18

Table B Income Elasticity Estimates of ϵ_M (Frank, p 135)	
Good/Service	Price Elasticity
Automobiles	2.46
Furniture	1.48
Restaurant Meals	1.4
Water	1.02
Gasoline/oil	.48
Electricity	.20
Margarine	-2
Pork Products	-.2
Public Transport	-.36

Table C Cross Elasticity Estimates of ϵ_{XY} (Frank, p 136)		
Good/Service	wrt ΔP	Price Elasticity
Butter	Margarine	+.81
Margarine	Butter	+.67
Natural gas	Fuel oil	+.44
Beef	Pork	+.28
Electricity	Natural gas	+.2
Entertainment	Food	-.72
Cereals	Fresh fish	-.87

Price, income and cross elasticity can be used to estimate probable effects of changes in price and incomes on buyers' behavior.

Problems:

1. If Mosquero Power and Light Company want to raise the price of electricity from \$.04 to \$.044, what would you estimate the change in the amount of electricity (that the buyers would "demand") would be?

$$\epsilon_p = \frac{\% \Delta Q_x}{\% \Delta P_x} = -1.2 = \frac{\% \Delta Q_x}{+10\%}$$

$$\% \Delta Q_x = -12\%$$

- 1.b) Briefly describe the changes in purchasing patterns of buyers (and potential buyers).

2. If the price of natural gas were expected to increase by 5% due to a new tax levied by the Canadian government, what would you expect to happen to the demand for electricity?

$$\epsilon_{XY} = \frac{\% \Delta Q_x}{\% \Delta P_y} = +.2 = \frac{\% \Delta Q_x}{+5\%}$$

$$\% \Delta Q_x = +1\%$$

3. "Butter is a better substitute for margarine than margarine is for butter." What does this mean and what evidence supports the statement?