"Simple Exchange"

Reynolds

I. Introduction

Neoclassical economics uses the criteria of Pareto Optimality to evaluate individual choices and social events with respect to the problems of "maximizing individual utility (or welfare)" and the performance of an economics system or institution in allocating resources (and goods) to their highest valued use.

Individuals know their preferences and presumably will act in their self-interest. They will make decisions that will improve their welfare (increase utility) or will do things that make them "no worse off." This means that all voluntary exchanges of goods with exclusive property rights among individuals will be Pareto improvements (movement toward a Pareto Optimal equilibrium).

Refresher:

Pareto efficiency is the condition where there are no alternatives that will improve the welfare of at least one person without making some one else "worse off."

Pareto superior is an alternative where the welfare of at least one person is greater and no one in any "worse off."

A Pareto improvement is a choice that improves the welfare of at least one person and no one is any worse off.

Pareto potential is a circumstance where at least one person is "better off" and at least on other person is worse off but the person who is better off (the "winner") can hypothetically compensate the "loser" and still be better off.

II. An Edgeworth Box or Edgeworth-Bowley Box

Francis Ysidro Edgeworth (1845-1926) provided economic theory with an important tool to explain the benefits of exchange in a market economy. He began with a given output of two goods. More than two goods can be considered but the model becomes multidimensional. The interactions between two goods and two individuals can be shown on a two-dimensional graph.

A. Two goods X and Y have been produced. This can be show by the width and height of the Edgeworth Box. In Figure 1, the length of the box represents the output of good Xebecs (in our example, Q_X = 150 units). The height of the box in Figure 1 represents the quantity of good Yawls (Q_Y = 95 units in the example).

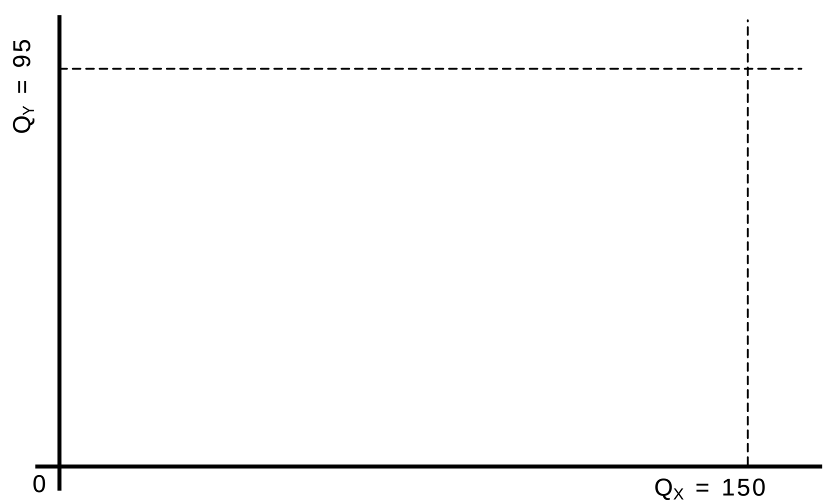


Figure 1

B. The 150 units of X (xebecs) and 95 units of Y (yawls) are divided between two individuals, Ann (A) and Bob (B). The amount of xebecs that Ann gets is shown as a distance along the X-axis (along the bottom) from left to right. The amount that Bob gets will be shown along the X-axis (along the top) from right to left.

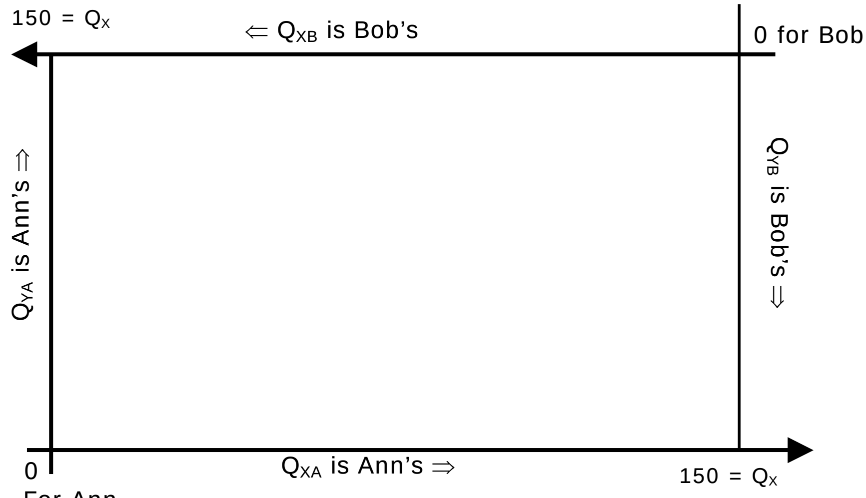


Figure 1.a

C. The distribution of good Y will be show along the vertical axis. The amount of good Y that Ann gets is measured along the left vertical axis from bottom to top. Bob's Y is measured along the right vertical axis from top to bottom in Figure 1.a.

D. In Figure 1.b the distribution of goods X (xebecs) and Y (yawls) can be shown by a point inside the Edgeworth Box.

1. If there are 150 units of good X and Ann gets 100 units, Bob must get 50 units (150-100=50). This distribution can be shown as line RR"

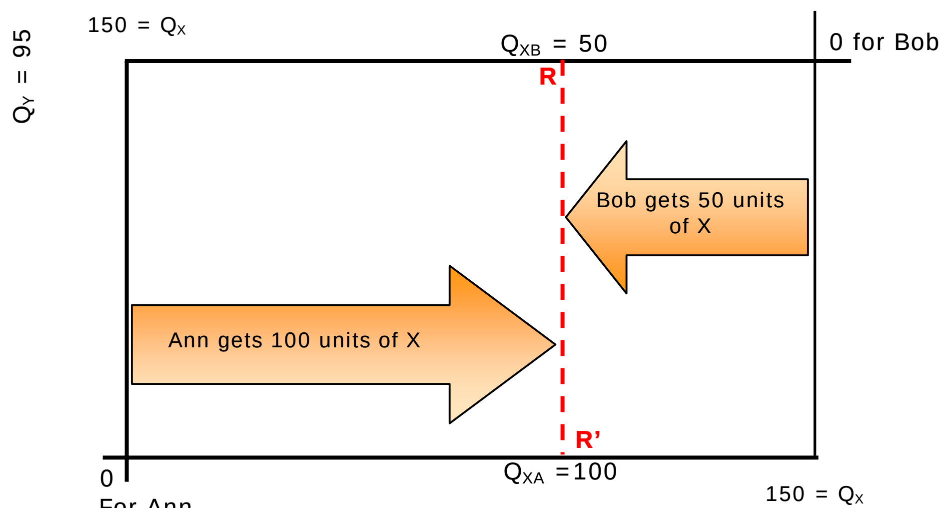


Figure 1.b

2. The 95 units of good Y are divided between Ann and Bob. If Ann gets 50 units of good Y, Bob will get 45 unit of Y (95-50=45). This distribution can be shown as line FF' in Figure 1.c.

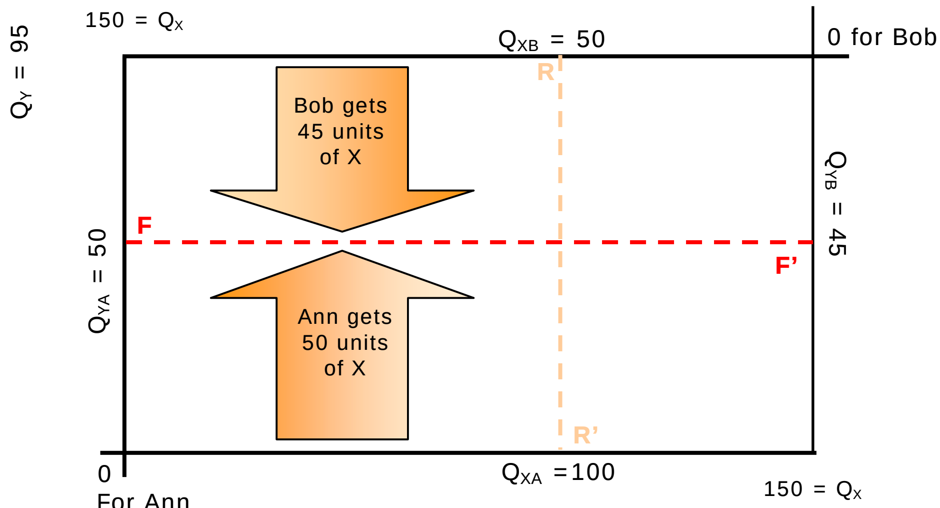


Figure 1.c

3. In Figure 1.d, point G represents a distribution where Ann gets 100 units of good X (Q_{XAG}) and 50 units of good Y (Q_{YAG}) while Bob gets 50 units of good X (Q_{XBG})and 45 units of good Y (Q_{YBG}).

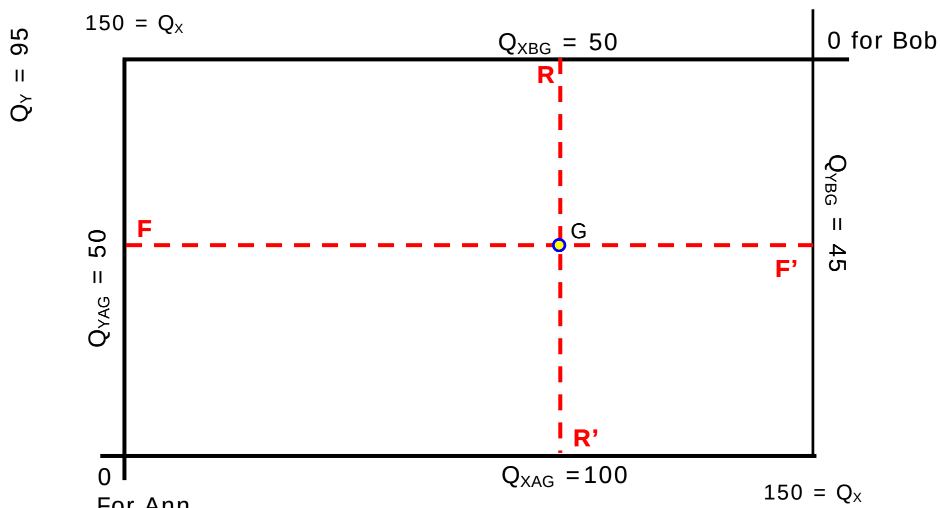


Figure 1.d

4. In Figure 1.f, point α shows another distribution of goods X and Y between Bob and Ann.

The distribution of goods between Bob and Ann can be accomplished by exchange. If Ann traded 25 units of good Y to Bob for 30 units of good Y, the distribution would move from point G to point α.

At α, Ann has Q_{XA} = 130, Q_{YA} = 25. Bob will have Q_{XB} = 20 and Q_{YB} = 70.

The distributions at points G and α indicate nothing about the relative welfare (utility) of either Bob or Ann.

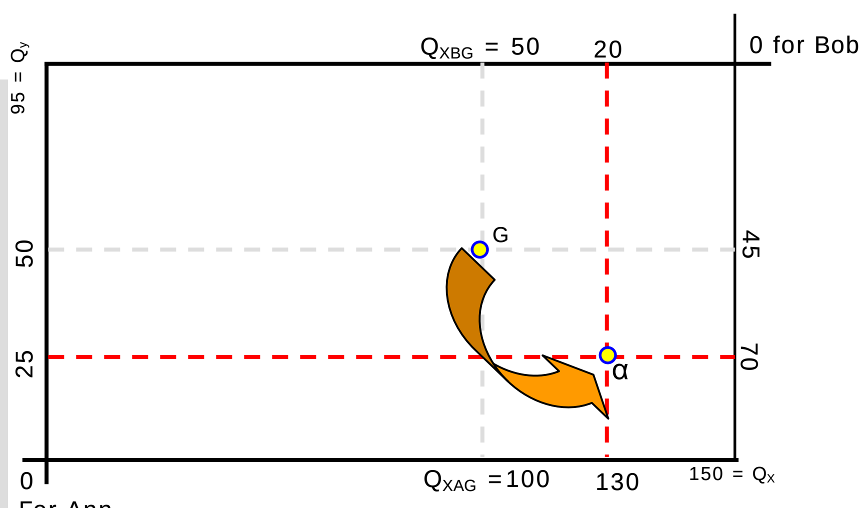


Figure 1.f

E. Individual Utility function can be shown as indifference maps. Ann's preferences are shown in Figure 2.A. Bob's indifference curves are shown in Figure 2.B. Remember that "normally shaped" indifference curves are convex to origin, every where dense, more is preferred to less and they do not touch or intersect.

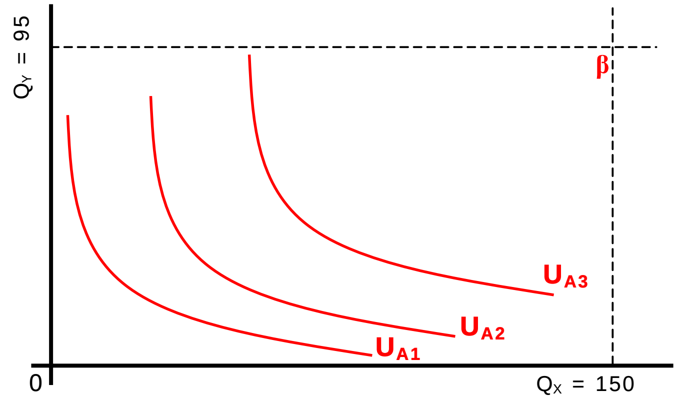


Figure 2.A

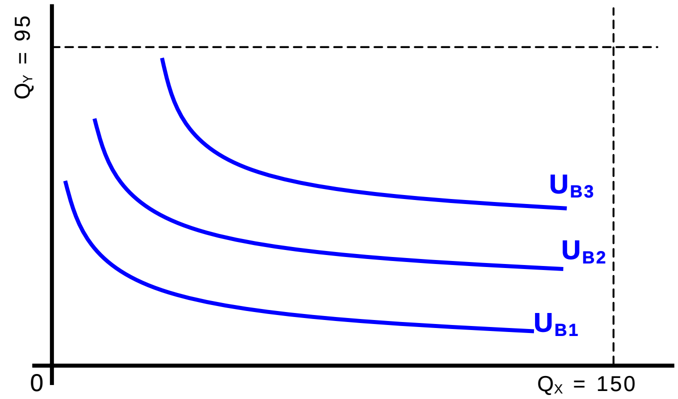


Figure 2.B

Ann and Bob do not necessarily have the same preferences, i.e. "like" xebecs and yawls the same. They have different shaped indifference curves. Since indifference curves are "everywhere dense," (the individual can rank any market basket) each individual has an infinite number of indifference curves. In Figure 2.A and 2.B, only 3 indifference curves have been displayed for each person.

III. Edgeworth Box and Exchange

The effects of any distribution of goods on Ann and Bob can be shown in an Edgeworth Box. In Figure 3.a both Ann and Bob's indifference maps are shown in "goods space." The quantity of output of the goods determines the size of the Edgeworth Box. In Figure 3.a the length or width of the box represents 150 units of good X.

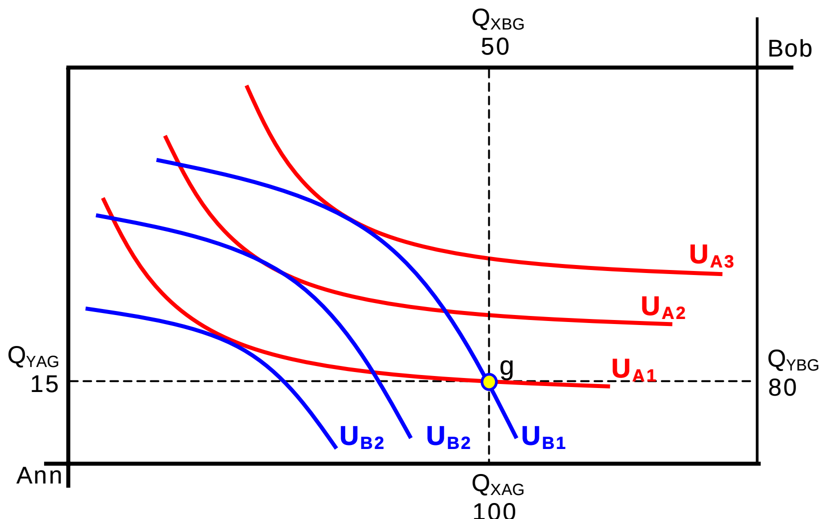


Figure 3.a

Bob and Ann's indifference curves can be shown in the same product space. QX is measured along the horizontal axis. What Ann does not get Bob does, $Q_{XA}+Q_{XB}=Q_X$. The same is true for good Y measured along the vertical axis. Ann's share of good X and Y and indifference curve (level of satisfaction or utility) are measure from the lower left hand corner of the box. Bob's share of the goods and indifference curves are measured from the upper right hand corner.

There are an infinite number of indifference curves for both Ann and Bob so it is possible to find 3 sets of curves that are tangent.

Bob's indifference map has been inverted and positioned over Ann's map.

An increase (decrease) in the output of good X will increase (decrease) the box's width. The height of the box is determined by the quantity of good Y. There are an infinite number of Edgeworth Boxes, one for every output combination or market basket represented by a point on or inside a production possibilities frontier (PPF). General equilibrium theory can be used to describe the optimal point on a PPF and its accompanying Edgeworth box. For now, the output of good X and Y are given.

In Figure 3.a, Q_X = 150, Q_Y = 95. Point g represents one possible distribution of the goods available; Ann get Q_{XAg} = 100 and Q_{YAg} = 15, Bob gets Q_{XBg}=50 and Q_{YBg}= 80.

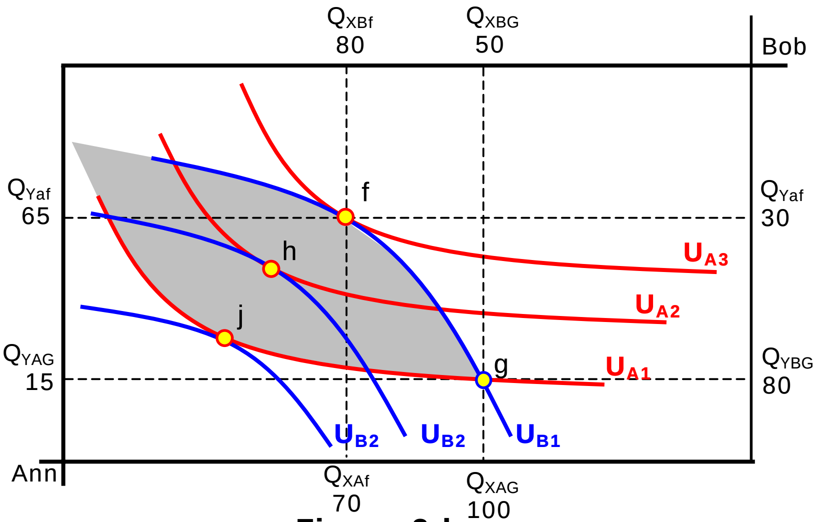


Figure 3.b

In Figure 3.b there are 150 units of good X (Q_X = 150) and 95 units of good Y (Q_Y = 95) available. This output combination is divided between Ann and Bob. At point g, Ann gets 100 units of X (Bob will get the remaining 50 units) and 15 units of good Y (Bob would get 80 units of Y).

Given this allocation, Ann will be on indifference curve U_{A1} and Bob will be on U_{B1}. Both Ann and Bob could be "better off" or reach higher indifference curves.

Ann would like to trade 30 units of good X to Bob for 50 units of good Y. This would increase Ann's utility and Bob would be no "worse off."

Given this distribution of goods X and Y (at point g in Figure 3.b), Ann's level of utility is indicated by U_{A1} and Bob's is represented by U_{B1}.

Both Ann and Bob can be "better off" i.e. both can reach higher indifference curves. Ann would like to trade 30 units so good X to Bob for 50 units of good Y. This would put Ann on indifference curve U_{A3} at point f, i.e. Ann is "better off" than at point g. At point f, Bob is no worse off than he was at point g; Bob is on his same indifference curve, U_{B1}.

Bob would like to trade 15 units of good Y for 65 unit of good X. this exchange would move the allocation to point j. Bob would move from U_{B1} to U_{B3} (be "better off") while Ann would remain on U_{A1} and is "no worse off."

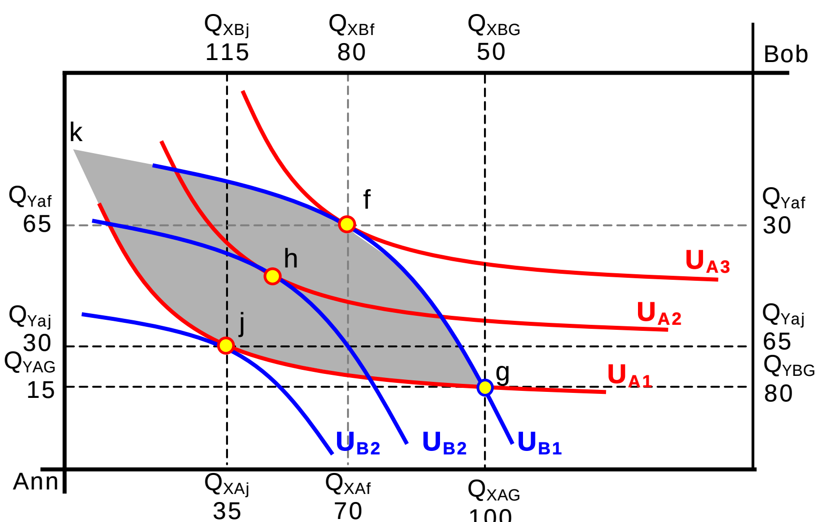


Figure 3.d

In Figure 3.d, Bob would like to trade 15 units of good Y for 65 units of good X. This will move Bob to a higher indifference curve and leave Ann on her original indifference curve; Bob is "better off" and Ann is "no worse off."

While Bob prefers to move from the allocation at point g to that at point j, Ann's preference is to move to the distribution represented by point f. Both Ann and Bob have an incentive to exchange with the other.

Any allocation that lies in the shaded area is Pareto superior to that at point g.

A. Reservation Prices

In the example, the maximum amount of good of good X that Ann will trade for good Y is 4.33 units. The maximum amount of good Y that Bob will trade for good X is 1.67 units of good Y. The exchange to move Ann and Bob from point g to point f represents Bob's reservation price (the maximum price that Bob will pay for good X) while the move from g to j represents Ann's reservation price (the maximum price in real terms that Ann will pay or trade for good Y)

B. Ann and Bob will be willing to exchange goods X and Y. Any exchange that moves them to a point in the shaded area (between U_{A1} and U_{B1}, area gfkj) in Figure 3.d will be a Pareto improvement.

C. Movement toward Pareto Efficiency

If the distribution were at point g in Figure 3.e and neither Ann nor Bob new the "reservation" price of the other, they might agree to move to

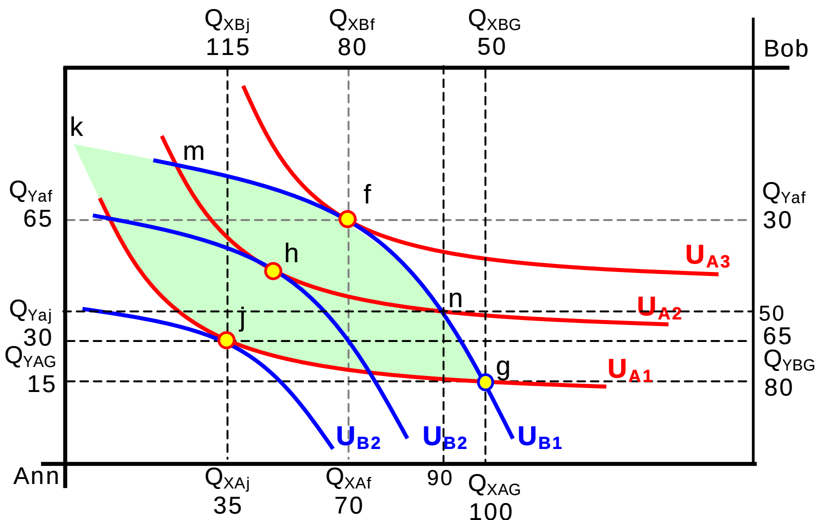


Figure 3.e

In Figure 3.d, Bob would like to trade 15 units of good Y for 65 units of good X. This will move Bob to a higher indifference curve and leave Ann on her original indifference curve; Bob is "better off" and Ann is "no worse off."

While Bob prefers to move from the allocation at point g to that at point j, Ann's preference is to move to the distribution represented by point f. Both Ann and Bob have an incentive to exchange with the other.

Any allocation that lies in the shaded area is Pareto superior to that at point g.

point n. At point g, Ann got 100 units of X and 15 of Y (Bob got 50 of X and 80 of Y). This distribution has given Ann "too much" of good X and Bob "too much" of good Y. The move to point n would involve Ann trading 10 units of good X for 30 units of good Y. Bob trades 30 unit of good Y for 10 units of good X. Bob is no "worse off," he's on the same indifference curve (U_{B1}). Ann is "better off," she is on a higher indifference curve (U_{A2} is a greater level of utility than U_{A1}).

Once the distribution is at point n (Ann has 90 units of X and 45 units of Y; Bob has 60 units of X and 50 of Y), there are additional gains that can be made by further exchanges. The distributions inside the area nfmh will result in Ann or Bob or both Ann and Bob being "better off" with no one any worse off.

There is an incentive to engage in exchange until Bob's marginal rate of substitution (MRS^B) is equal to Ann's marginal rate of substitution (MRS^A). Remember the MRS (slope of the indifference curve) is a measure of the willingness to exchange one good for the other. When the MRS of the two individuals are equal (MRS_A=MRS_B) there will be no incentive for at least one of the parties to exchange. Whether Bob and Ann exchange to move from point g to point j, h or f in Figure 3.e, once they get to a point where their respective indifference curves are tangent, there is no incentive to trade. If they were at point j, any trade would make one or both parties "worse off." An exchange that moves the distribution from point j to h will make Ann better off but will make Bob worse off. Once exchange has equated the MRS's, no additional gains from trade can be made without making one or both "worse off." Pareto optimality or efficiency is consistent with all distributions where the MRS_A = MRS_B.

D. All Pareto efficient distributions of the 150 units of good X and 95 of good Y are where Ann's indifference curve is tangent to Bob's. Since there are an infinite number of these distributions, a "contract curve" is constructed from one corner of the box to the other along all points where $MRS_A = MRS_B$. The contract curve is shown in Figure 3.f as the line αβ. At point α, Ann's utility is 0, Bob gets all of both goods. At point β, Ann gets all of both goods and Bob gets none.

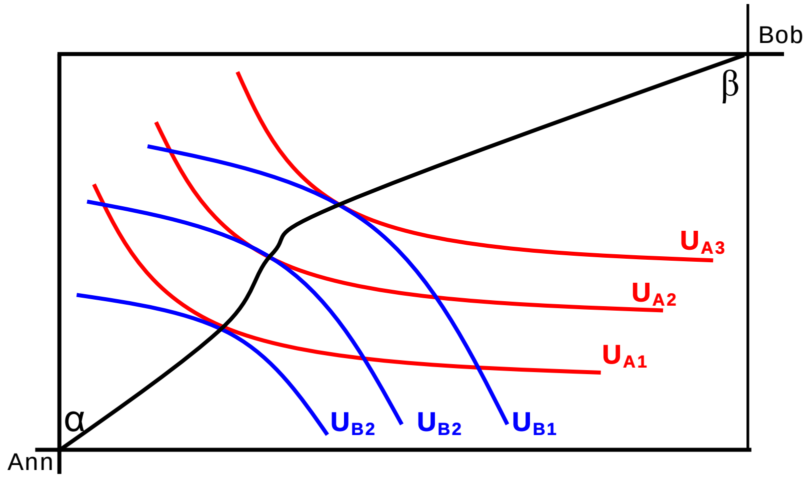
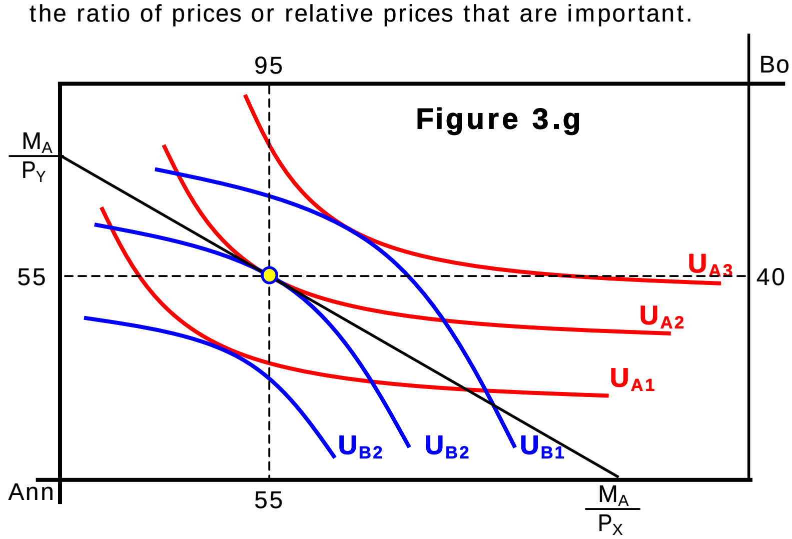


Figure 3.f

E. Barter is often associate with significant transaction costs and the problem of "double coincidence of wants." Barter often requires significant effort to find and negotiate trades. If you find someone who has what you want, they may not want what you have. If you have potatoes and want bread and have found some one who has bread but wants apples, you may have to make many trades to get what you want. Money can be used as a "medium of exchange" and will facilitate exchanges with lower transaction costs.

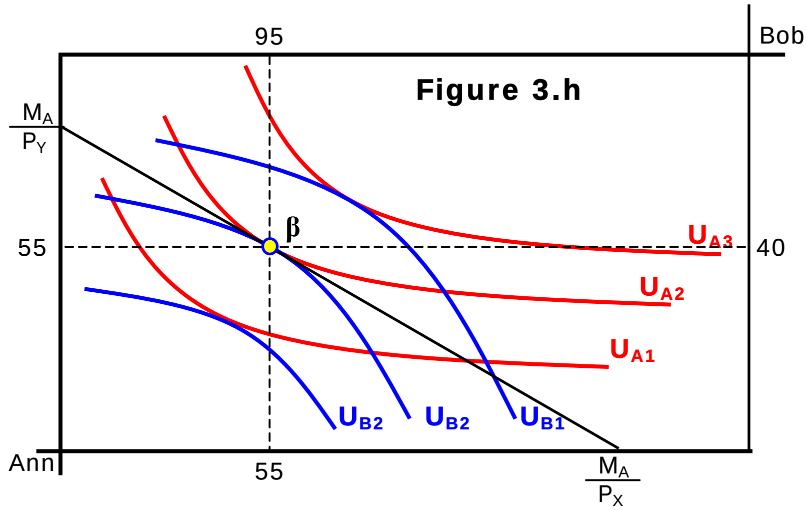
F. With the introduction of money, the relative incomes of the parties and the relative prices become and issue. Ignoring the problem of whether the "right" relative amounts of goods X and Y are produced, it is possible to determine the appropriate set of relative prices of the two goods given the initial income distribution between Ann and Bob.

1. In the simplified version no price discrimination is allowed. Bob and Ann must pay the same prices for each of the goods.
2. The total income is irrelevant. A greater total income just requires higher prices. Lower total income will result in lower prices. It is the ratio of prices or relative prices that are important.



In Figure 3.g, Ann and Bob are confronted with a set of relative prices and a total income that is distributed between them. In the case shown in Figure 3.g they do not get equal incomes. Ann's income is less than Bob's.

If they are both confronted with the same prices of goods X and Y (P_X and P_Y), they will try to maximize their utilities given M, P_X and P_Y. This will



occur at point β Figure 3.h. There are a number of conditions that have been met.

1. Ann and Bob have exhausted all gains from exchange; the slopes of their indifference curves are equal so, $MRS^A = MRS_B$.
2. Ann is maximizing her utility given her income share and the set of relative prices;

$$MRS^A = \left|-\frac{P_X}{P_y}\right|, \text{ and } M^A = P_X Q_X^A + P_Y Q_Y^A$$

3. Bob is maximizing his utility:

$$MRS^B = \left|-\frac{p_X}{P_Y}\right|, \text{ and } M^B = P_X Q_X^B + P_Y Q_Y^B$$

4. Ann and Bob are paying the same price for each good
5. $M^A + M^B = M$, (the total income is divided between Ann and Bob
6. The quantity of each good purchased by Ann and the quantity purchased by Bob are equal to the total output;

$$Q_X = Q_X^A + Q_Y^B \text{ , and } Q_Y = Q_Y^A + Q_Y^B$$

G. Summary:

The Edgeworth box shows that voluntary market exchange of goods with nonattenuated property rights and a price system with no price discrimination will result in a "correct" set of relative prices and an optimal distribution of goods given a set of goods "that have been produced" and a given distribution of income (relative incomes).

$$MRS^A = \left|-\frac{P_X}{P_Y}\right| = MRS^B,$$

$$Q_X^A + Q_X^B = Q_X, \text{ and } Q_Y^A + Q_Y^B = Q_Y,$$

$$\text{and } M^A + M^B = M,$$

$$P_X Q_X^A + P_y Q_Y^A = M^A, \; P_X Q_X^B + P_Y Q_Y^B$$

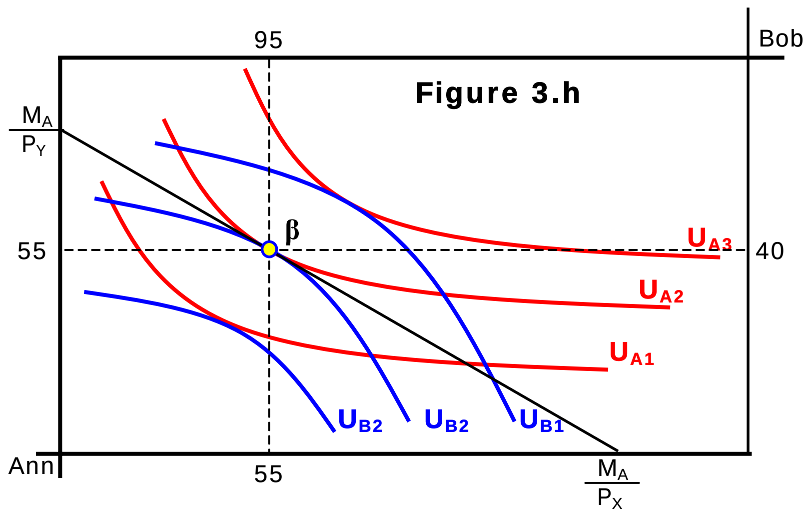


Figure 3.h
95
Bob
M_A / P_Y
β
55
40
U_A3
U_A2
U_A1
U_B2
U_B2
U_B1
Ann
55
M_A / P_X

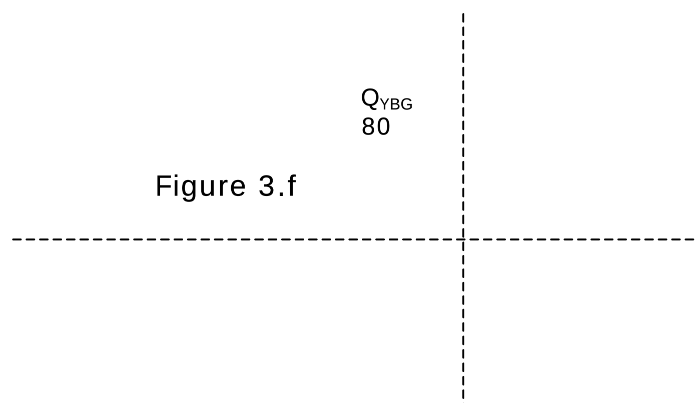

Q_YBG
80

Figure 3.f