

BASIC MICROECONOMICS: AN OUTLINE MATH REVIEW

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One of the important tasks in economics is the evaluation of alternatives to determine which best satisfies given objectives or goals. In order to do this it is often desirable determine cause and effect relationships and to quantify variables. Mathematics is a powerful tool that aids both these tasks. It is impossible to do economic analysis without some elementary understanding of basic math tools.

1. Mathematics as a Tool

Mathematics is a very precise language that is useful in expressing causal relationships between related variables. Since microeconomics is the study of the relationships between resources and the production of goods that are used to satisfy wants, mathematics is indispensable. When decisions are made about the allocation of resources, it is desirable to be able to express how a change in one input will alter the output and ultimately change the utility of individuals.

Relationships, Equations, Graphs and Tables

A relationship between two or more variables can be expressed as an equation, table, and graph or in a narrative form. Narratives are often difficult to understand in precise ways. Tables, graphs and equations are efficient methods of quickly and clearly expressing relationships.

Equations and graphs are used frequently in economics because they present data in such a way that patterns of the relationships are easy to visualize. Equations and graphs usually represent data about relationships as continuous functions. Tables contain discrete data. Data is said to be discrete when there is some unit that is indivisible. For example, people are counted 1, 2, 3, ... This is a discrete set. Gallons of gasoline can be counted 1.1, 1.2 and we know that 1.15 lies between 1.1 and 1.2. Further 1.153 lies between 1.15 and 1.2. This data is continuous.

Tables are less complete than equations or graphs. In a table, only the information in a row or column exists. There is no information about what happens between rows or columns. It is more difficult to recognize and visualize patterns in data that is presented in tables.

Equations and Graphs

An equation expresses a relationship between two (or more) variables. The value of the dependent variable is determined by the structure of the equation and the value of (each of) the independent variables. A linear relationship is expressed:

$$Y = b + mX$$

Y is the dependent variable. The value that Y takes on is determined by the structure of the equation (b, and + m) and the value of X which is the independent variable. The value of the independent variable, X is determined outside the equation. "b" is a parameter that is called the "Y intercept." When X is zero (X = 0), then the value of Y equals b. "m" is the slope of the equation. It measures the change in Y caused by a one unit change in X.

As an exercise, assign values to the parameters of the equation;

$$b = 5$$

$$m = 2$$

$$\text{Then } Y = 5 + 2X$$

$$\text{When } X = 0, \text{ then } Y = 5$$

$$X = 4, \text{ then } Y = 13$$

For any value of X the value of Y is determined. This information can be shown in Table 1.

Table 1	
$Y = 5 + 2X$	
X	Y
0	5
1	7
2	9
3	11



The information can also be graphed. Note that when $X = 0$, $Y = 5$. When $X = 1$, $Y = 7$. The slope of the line is $+2$. For every increase in the value of X by 1, Y will increase by 2.

For the equation $Y = 9 - 2X$ the graph can be shown;

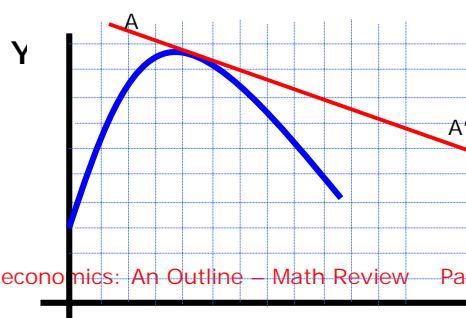


When $X = 0$, then $Y = 9$. For $X = 4.5$ then $Y = 0$. The equation $Y = 9 - 2X$ has a negative slope. As X increases (by 1), the value of Y will decrease (by -2).

The value of X is determined outside the equation. It is exogenous. The value of Y is endogenous or determined inside the equation.

Relationships can be nonlinear. They might take the form $Y = b + mX - hX^2$.

In this case the relationship between X and Y will change at different values of X . The rate of change in Y (caused by changes in X) can be visualized as a tangent at each point on the function. See line AA' . A derivative is the



rate of change in Y as changes in X approach 0.

Functional Notation

A simple way to express a relationship is to say **“the value of the dependent variable Y is determined by the value of the independent variable X.”** Alternatively, we might say $Y = f(X)$ or Y equals a function of X. When there are several independent variables the expression would be $Y = f(X, Z, D, \dots)$. This means that the value of Y is determined by the values of X, Z, D, and other things. The three dots (...) usually means there are other variables but they are not shown.

Slope of a line

The slope of a line (m) is defined as the rise over the

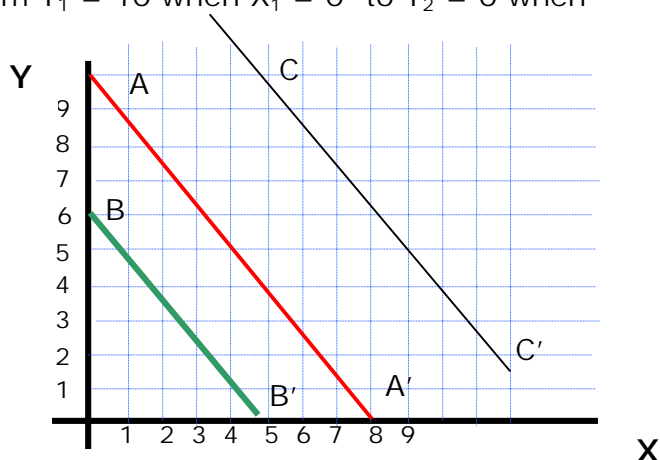
$$m = \frac{\text{Rise}}{\text{Run}} \quad \text{run};$$

The rise is the change in Y and the run is the change in X. for the line AA' in the graph to the right, the line decreases from $Y_1 = 10$ when $X_1 = 0$ to $Y_2 = 0$ when $X_2 = 8$. The rise is -10 ($Y_2 - Y_1 = \text{rise}$) or $0 - 10 = -10$. The run is the distance from $X_1 = 0$ to $X_2 = 8$ ($X_2 - X_1 = 8$).

The slope of this line is $-10/8$ or -1.25 . As X increases by 1, the value of Y will decrease by 1.25.

Shifts of Functions

If the value of the intercept changes, the function will “shift.” In the graph to the right the equation is $Y = 10 - 1.25X$. A change in the intercept from 10 to 6 will “shift” (or generate a new function) that is depicted by line BB'. And increase in the intercept will “shift” the function out, line CC'.



2. Statistics as a Tool

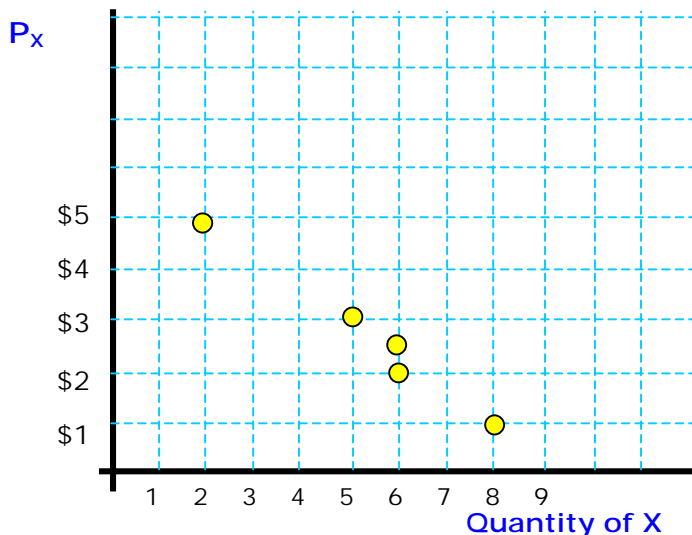
In microeconomics, models are used to explain the relationships between variables. Consider a case where Susan buys coffee. A model that might explain Susan's behavior would include several variables. These might include;

- The price of coffee (P_X)
- Susan's income (Y)
- The day of the week
- Time of day
- The prices of alternatives (cola, hot choc, etc) (P_R)
- Susan's religion
- Susan's health
- Susan' age

If P_X , Y, P_R and preferences were all changing at the same time, we would use a multivariate analysis called **“multiple regression.”** For simplicity, we have been lucky enough to find a period where only P_X has changed, Y, P_R and preferences have remained unchanged over the period in which we observe Susan's purchases. The price of coffee (P_X) is constant over the week and is only changed

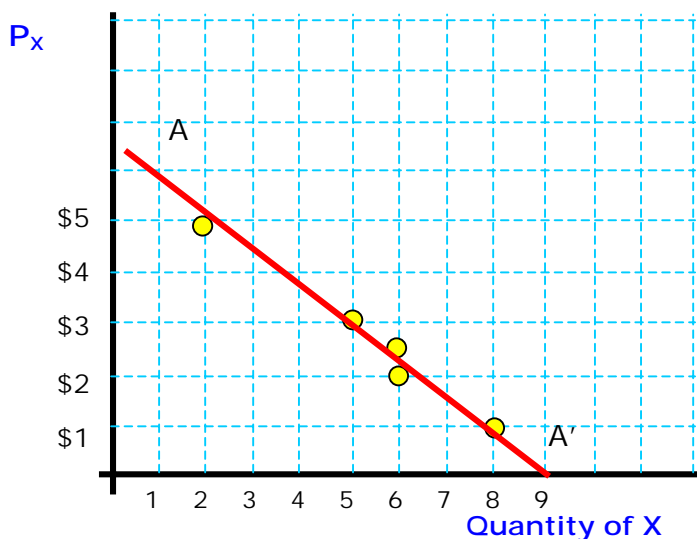
at the beginning of each week. Susan's behavior is observed each week, so the model we are constructing is for that time period. In an actual case we would need many more observations than are used in the example. The observations of P_x and the quantity that Susan buys each week is presented in Table 2

Table 2 Susan's Purchases		
Week	Price	Quantity
1	\$1	8
2	\$2	6
3	\$5	2
4	\$3	5
5	\$2.50	6



While there are few observations, a pattern emerges. At higher prices, a smaller quantity is purchased.

There is a relationship between price and quantity: $Q = f(P)$. The relationship is not perfect, but there is a relationship. Statistics provides a tool to summarize the relationship, it is called "ordinary least squares" (OLSQ, or OLS). This tool fits a line that minimizes the variance (or squared distance) that each data point lies off the line. In the graph below the relationship estimated through OLSQ is line AA'.



Line AA' is an estimate of the relationship between price and quantity purchased. The values of the intercept and the slope are estimated. Additional information can be calculated on the standard error and the robustness of estimate. There is another related technique, "multiple regression" that is used to relate several independent variables to the dependent variable. This technique allows the analysis of several variables and describes how each influences

the dependent variable.

3. Ceteris Paribus

Economists function in a world where there are many variables that are constantly changing. It is impossible to determine the effects of changes in variables when several things change at once. To minimize this problem, economists try to isolate the effects on changes in one variable at a time. Your

auto mechanic and doctor do the same thing. If you took your car to a mechanic and told her that it did not run, you would like them to use *ceteris paribus*, i.e. change one thing at a time. The car might not run because:

- The computer was dead
- The battery was dead
- The fuel filter was plugged
- The fuel tank was empty
- The injectors were plugged
- The fuel pump was broken
- Or a variety of other problems.

You would like the mechanic to determine the most likely cause of the problem and change one thing at a time. If they changed everything and the car ran you wouldn't know what the problem was and it would cost a lot of money!