Demand and Elasticity

Elasticity is a way to measure the responsiveness of a dependent variable to changes in an independent variable.

Elasticity is defined as a ratio of the percentage change in a dependent variable to a percentage change in an independent variable.

$$\text{Elasticity} \equiv \frac{\text{percentage change of dependent variable}}{\text{Percentage change of independent variable}}$$

When: $Y = f(X)$

$$E \equiv \frac{\%\Delta Y}{\%\Delta X}$$

Examples of Elasticity

If, $Q_X = f\,(P_X, P_R, M, \ldots \#buyers)$,

1) It may be useful to know how a change in the price of good X (ΔP_X) will alter the quantity purchased at each price (ΔQ_X). Price elasticity (E_P, ε_p, η_P, e_P) measures this relationship.
2) Income elasticity is a measure of how a change in income (ΔM) will change the quantity (ΔQ_X) that will be purchased at each price.
3) Cross elasticity is a measure of how a change in the price of good Y (ΔP_Y) will change the quantity demanded of good X (ΔQ_X)

"Own" Price Elasticity of Demand

Price Elasticity of Demand (E_P) is a measure of how responsive buyers are to changes in the price of a good.

Price elasticity is determined by:

1) the slope of the demand function, $\left(\frac{\Delta Q}{\Delta P}\right)$

2) the location on the demand function (Q, P, or more accurately, $\frac{P}{Q}$)

Calculation of Price Elasticity

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$$E_P = \frac{\%\ \Delta\ \text{Quantity}}{\%\ \Delta\ \text{Price}} = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}}$$

By rearranging terms:

$$E_P = \frac{\Delta Q_X}{\Delta P_X} * \frac{P_X}{Q_X}$$

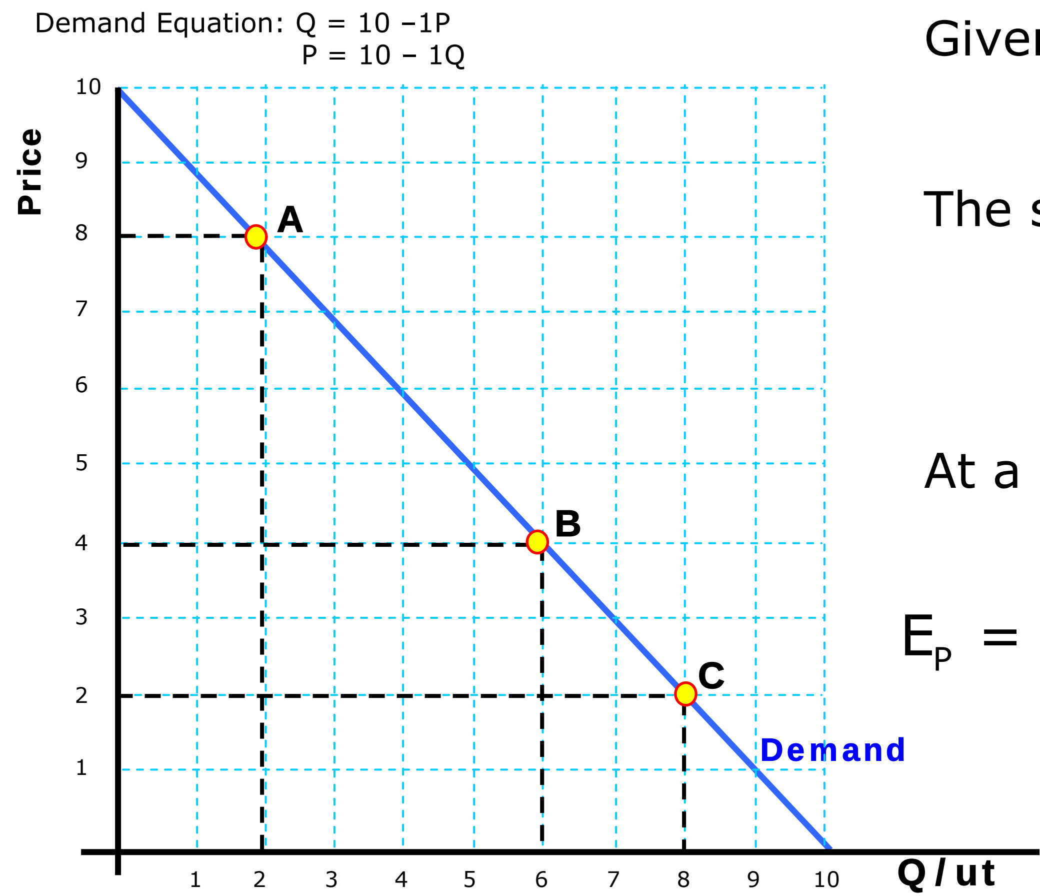
Calculation of Price Elasticity -

$$E_P = \frac{\Delta Q_X}{\Delta P_X} * \frac{P_X}{Q_X}$$

$\frac{\Delta Q_X}{\Delta P_X}$ is the slope of the demand function

$\frac{P_X}{Q_X}$ *is the location on the demand function*

Example of "Point" of Price Elasticity



Given the demand equation:
$Q = 10 - 1P$

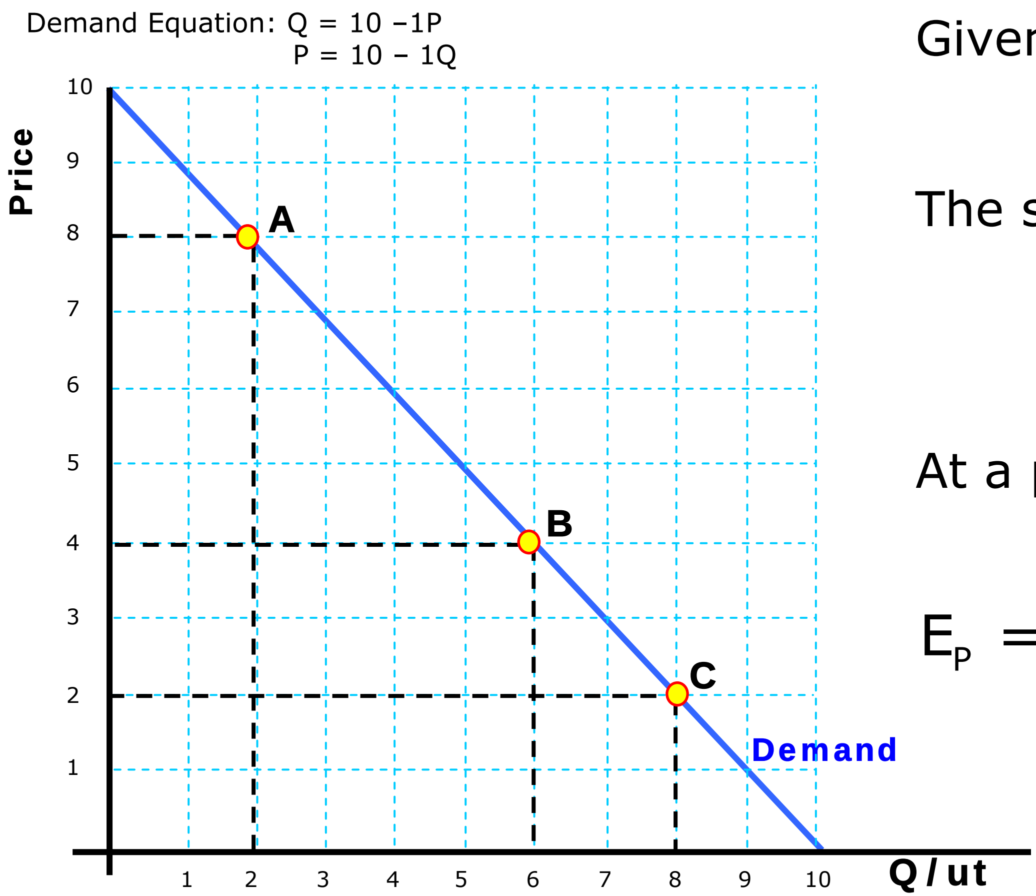
The slope of demand is:

$$\frac{\Delta Q_X}{\Delta P_X} = -1$$

At a price of $4, Q_X is 6 units

$$E_P = \frac{\Delta Q_X}{P_X} * \frac{P}{Q} = (-1)\left(\frac{4}{6}\right) = -.67$$

Example of "Point" of Price Elasticity



Given the demand equation:

$$Q = 10 - 1P$$

The slope of demand is:

$$\frac{\Delta Q_X}{\Delta P_X} = -1$$

At a price of \$8, Q_X is 2 units

$$E_P = \frac{\Delta Q_X}{P_X} * \frac{P}{Q} = (-1)\left(\frac{8}{2}\right) = -4$$

Point price elasticity is different at every price.

"Elastic" Demand

Point price elasticity is different at every price.

When buyers are responsive to ΔP;

- This is "ELASTIC"
- The $|\%\Delta Q_X| > |\%\Delta P_X|$
- $|E_P|>1$
- Total Revenue and price move in opposite directions.
- This occurs at "high prices"

Q = 10 – 1P
Demand, E_P and Total Revenue (TR)

Price	Quantity	E_P	TR
$0	10	0	0
$1	9	-0.11	$9
$2	8	-0.25	$16
$3	7	-0.43	$21
$4	6	-0.67	$24
$5	5	- 1.00	$25
$6	4	- 1.50	$24
$7	3	- 2.33	$21
$8	2	- 4.00	$16
$9	1	- 9.00	$9
$10	0	undefined	0

"Inelastic" Demand

Point price elasticity is different at every price.

When buyers are not responsive to ΔP;

- This is "INELASTIC"
- The $|\%\Delta Q_X| < |\%\Delta P_X|$
- $|E_P|<1$
- Total Revenue and price move in the same direction.
- This occurs at "low prices"

Q = 10 – 1P
Demand, E_P and Total Revenue (TR)

Price	Quantity	E_P	TR
$0	10	0	0
$1	9	-0.11	$9
$2	8	-0.25	$16
$3	7	-0.43	$21
$4	6	-0.67	$24
$5	5	- 1.00	$25
$6	4	- 1.50	$24
$7	3	- 2.33	$21
$8	2	- 4.00	$16
$9	1	- 9.00	$9
$10	0	undefined	0

"Unitary Elasticity" of Demand

Point price elasticity is different at every price.

When the percentage change in quantity demanded is equal to the percentage change in price;

- This is "Unitary" elasticity
- The $|\%\Delta Q_X| = |\%\Delta P_X|$
- $|E_P| = 1$
- Total Revenue is a maximum.

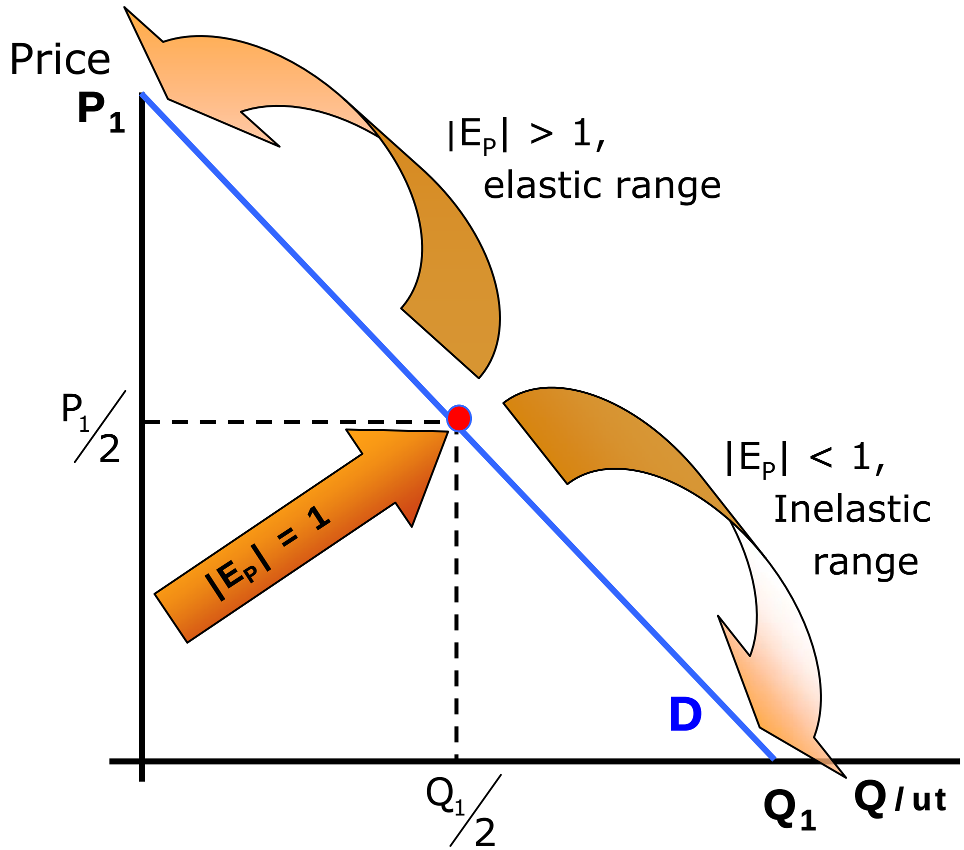
Q = 10 – 1P Demand, E_P and Total Revenue (TR)			
Price	Quantity	E_P	TR
$0	10	0	0
$1	9	-0.11	$9
$2	8	-0.25	$16
$3	7	-0.43	$21
$4	6	-0.67	$24
$5	5	- 1.00	$25
$6	4	- 1.50	$24
$7	3	- 2.33	$21
$8	2	- 4.00	$16
$9	1	- 9.00	$9
$10	0	undefined	0

"Midpoint" of Demand and Elasticity

Given any linear demand function, divide either the price or quantity intercept in half,

At that point on the demand function the coefficient of price elasticity will be unitary, (E_P = -1)

The top half ("high prices") of the demand function is "elastic" and the bottom half ("low prices") is "inelastic."



Revenue Functions

- Total Revenue (TR) is defined as price times quantity: $TR \equiv PQ$
- Average Revenue (AR) is defined as TR/Q or price: $$AR = \frac{TR}{Q} = \frac{PQ}{Q} = P$$
- Marginal Revenue (MR) is defined as the change in TR "caused" by a change in the price: $$MR \equiv \frac{\partial TR}{\partial Q} \approx \frac{\Delta TR}{\Delta Q}$$

Demand as $Q_X = f(P_X)$ or $P_X = f^{-1}(Q_X)$

Given a demand function:

$$Q_X = a - bP_X$$

$$Q_X - a = -bP_X$$

$$\frac{Q_X}{-b} - \frac{a}{-b} = P_X$$

$$P_X = \frac{a}{b} - \frac{1}{b}Q_X$$

Example:

$$Q_X = 30 - 4P_X$$

$$Q_X - 30 = -4P_X$$

$$\frac{Q_X}{-4} - \frac{30}{-4} = P_X$$

$$P_X = \frac{30}{4} - \frac{1}{4}Q_X$$

$$P_X = 7.5 - .25Q_X$$

Revenue and Demand - Arithmetic

$$P = g - mQ, \quad \text{(demand equation)}$$

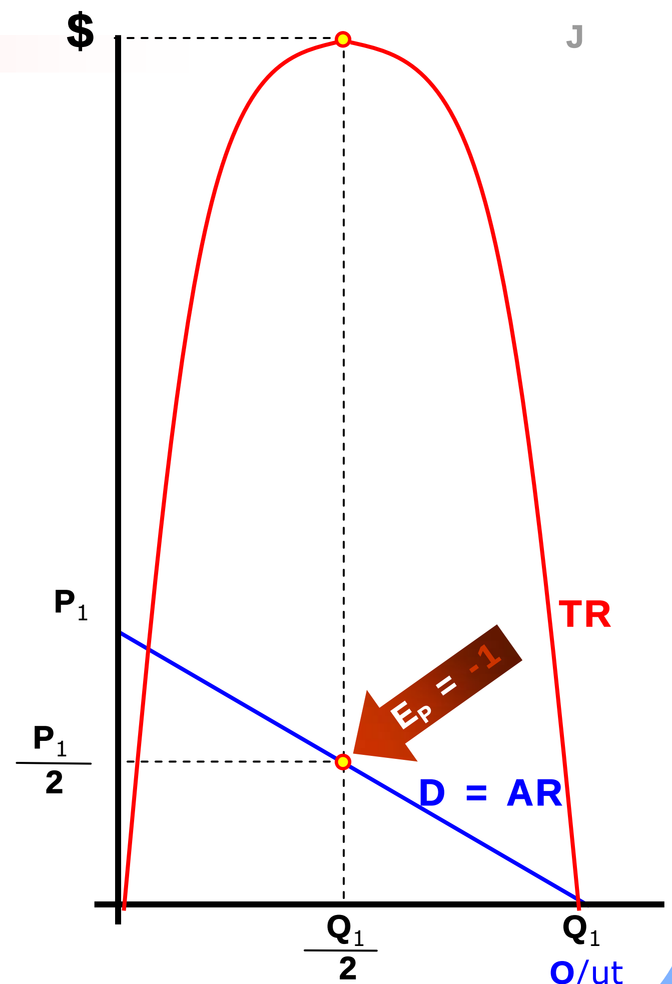
$$TR = PQ = (g - mQ)Q$$

$$TR = gQ - mQ^2$$

$$TR = gQ - mQ^2$$

$$AR = \frac{TR}{Q} = \frac{gQ - mQ^2}{Q} = g - mQ$$

$$P = g - mQ, \quad \text{(demand equation)}$$



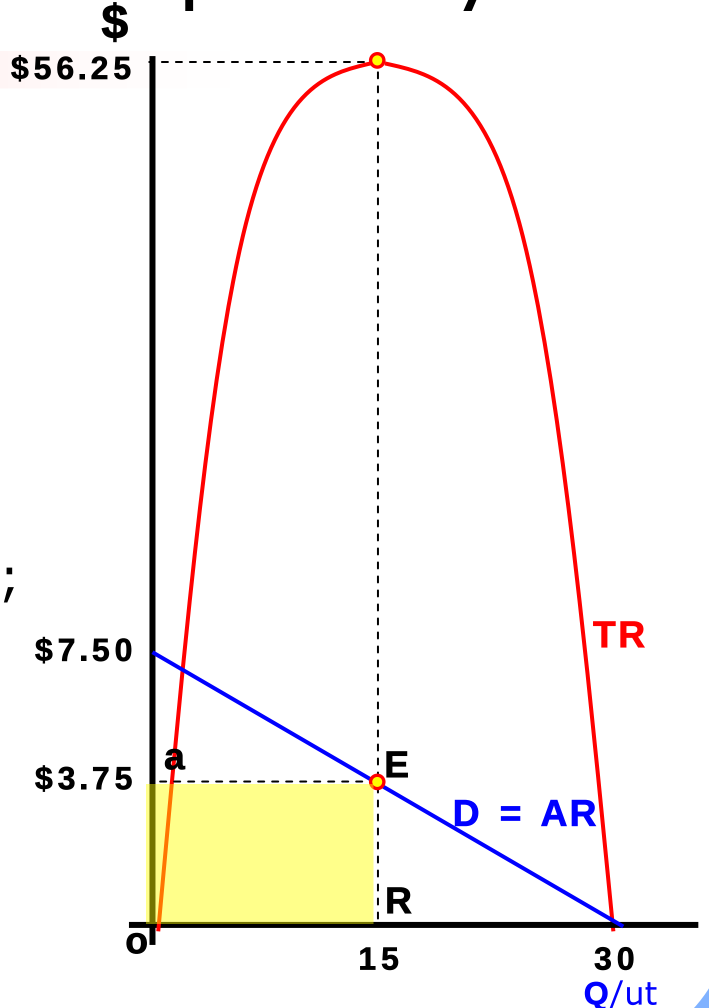
Revenue and Demand - Graphically

$$Q_X = 30 - 4P_X$$

$$P_X = 7.5 - .25Q_X$$

TR for 15 units of output at a price of \$3.75 can be shown as the height of the TR function at Q=15, TR=56.75

TR is also the area of the rectangle oREa; PQ = 3.75(15) = 56.25 (max. TR)



Revenue and Demand - Graphically

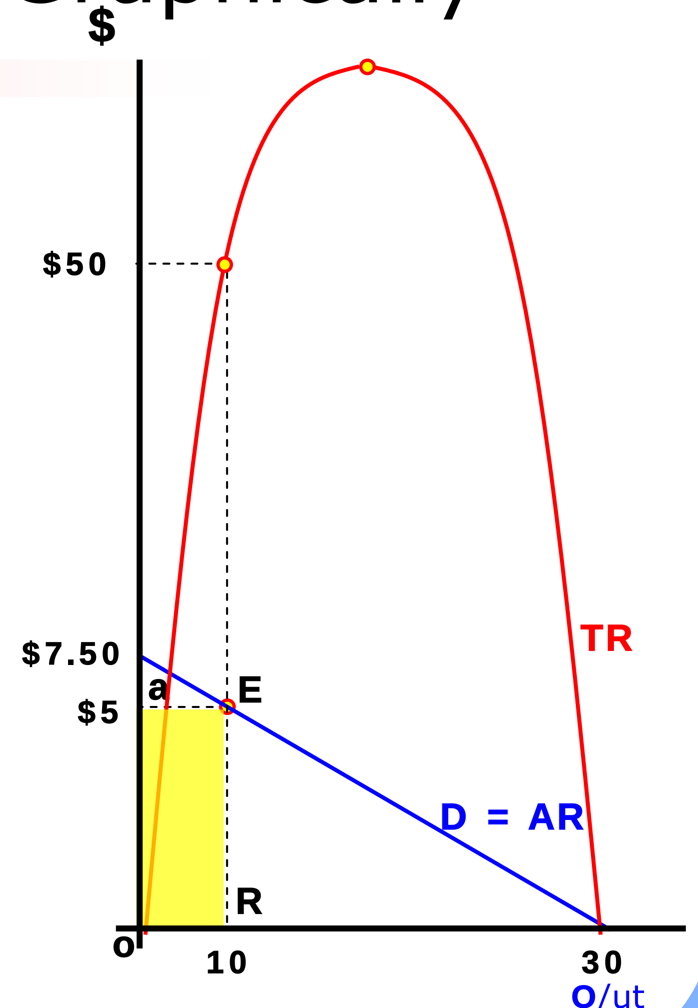
$Q_X = 30 - 4P_X$

$P_X = 7.5 - .25Q_X$

TR for 10 units of output at a price of \$5 can be shown as the height of the TR function ($TR=7.5Q-.25Q^2$) at Q=10, TR=50

TR is also the area of the rectangle oREa; PQ = 5(10) = 50

As the price of the good increased above the midpoint, TR decreased; the demand is "elastic" in this range.



Revenue and Demand - Graphically

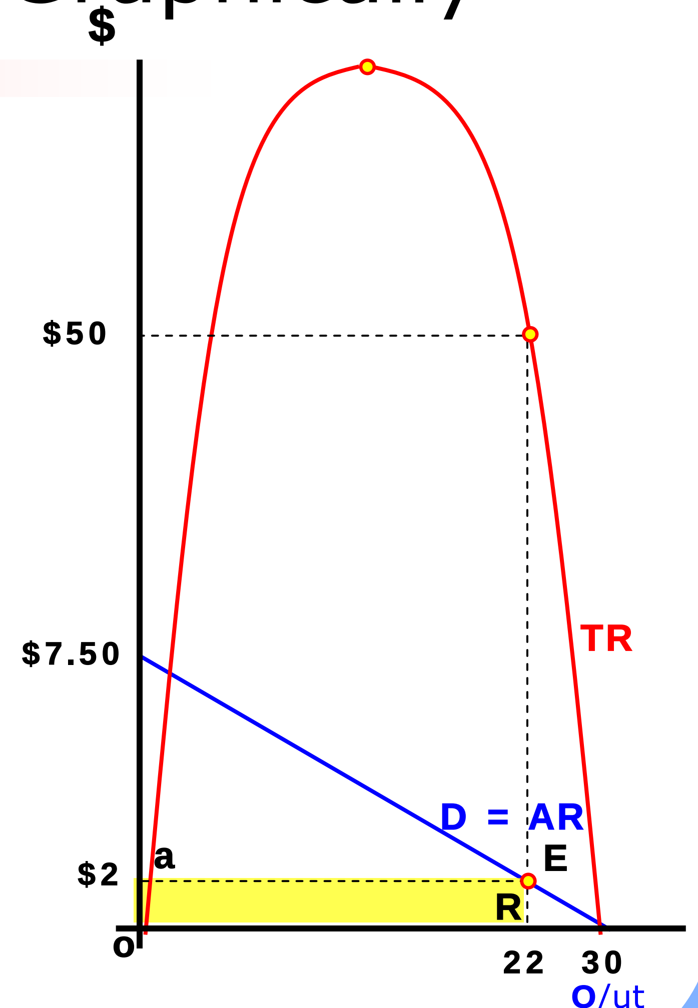
$$Q_X = 30 - 4P_X$$

$$P_X = 7.5 - .25Q_X$$

At a price of $2, TR can be shown as the height of the TR function ($TR=7.5Q-.25Q^2$) at Q=22, TR=44

TR is also the area of the rectangle oREa; PQ = 2(22) = 44

As the price of the good decreased above the midpoint, TR decreased; the demand is "inelastic" in this range.



Revenue and Demand - Table

Demand, TR, AR, MR and Elasticity					
Price	Q	TR	AR	MR	E_P
P = 7.7-.25Q	Q = 30-4P	$TR=7.5Q-.25Q^2$	AR=TR/Q = P	MR=dTR/dQ	$E_P=\%\Delta Q/\% P$
$0	30	0	$0	-7.5	0
$1	26	26	$1	-5.5	-.1539
$2	22	44	$2	-3.5	-.3636
$3	18	54	$3	-1.5	-.6667
$3.75	15	56.25	$3.75	0	-1
$4	14	56	$4	.5	-1.143
$5	10	50	$5	2.5	-2
$6	6	36	$6	4.5	-4
$7	2	14	$7	6.5	-14
$7.50	0	0	undefined	7.5	undefined

TR, AR, MR and Demand - Graphically

$$Q_X = 30 - 4P_X$$

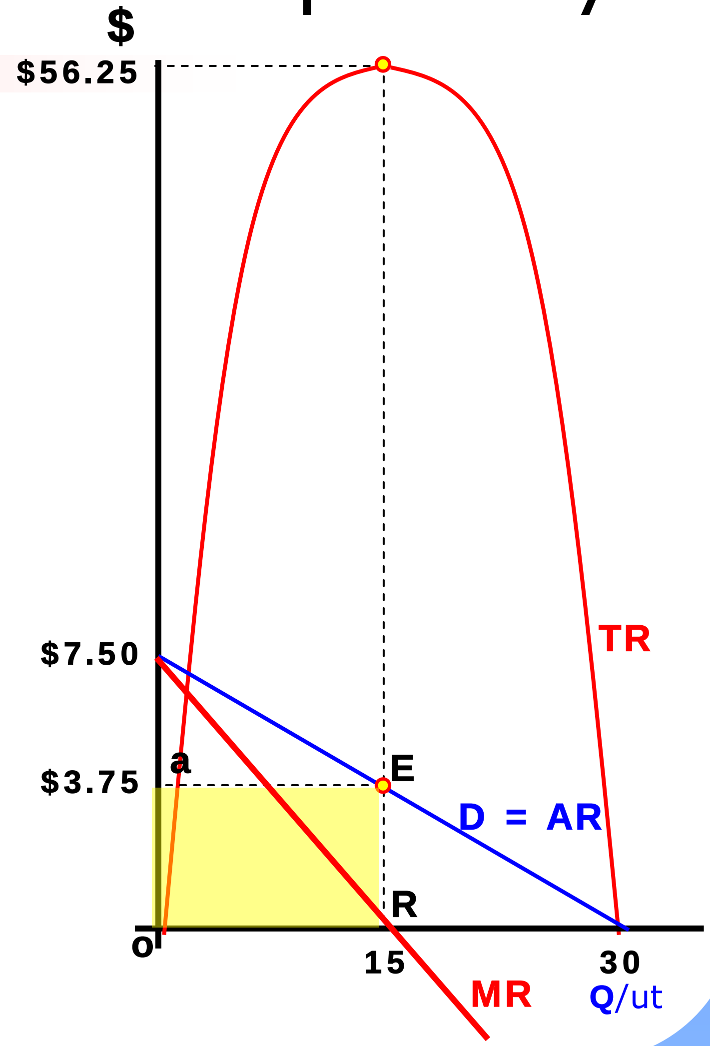
$$P_X = 7.5 - .25Q_X$$

$$TR = 7.5Q - .25Q^2$$

$$MR = 7.5 - .5Q_X$$

Where TR is a maximum;

- AR (demand) has a $E_P = -1$,
- It is a "midpoint" in AR for a linear demand function
- MR is 0, it is the Q intercept



Examples of E_P – long and short run

Selected Price Elasticities		
Item	Short Run E_P	Long Run E_P
Automobiles	-1.87	-2.24
Movies	-.087	-3.67
Medical Care	-.31	-.92
Gasoline	-.20	-.70

Interpretation of E_P

If the E_P for gasoline is -.7 in the long run, how will an increase in price alter quantity of sales and revenue?

If the price of gas goes up 20%, how much will the quantity demanded decline?

$$E_P = \frac{\%\Delta Q_X}{\%\Delta P_X}$$

$$E_P = -.7$$

$$\%\Delta P = +.2$$

$$-.7 = \frac{\%\Delta Q}{+.2}$$

$$\%\Delta Q = (-.7)(+.2) = -.14$$ (quantity demanded will decline by 14%)

Income Elasticity - E_M

Income elasticity is a measure of how responsive buyers purchases are to changes in their income.

$$E_M \equiv \frac{\%\Delta Q_X}{\%\Delta M} = \frac{\Delta Q_X}{\Delta M} * \frac{M}{Q}$$

The measure of income must remain “consistent.” Average disposable income, average gross income or net income might be used, it is necessary to be consistent as to which one you use.

Income Elasticity- Calculation

Given a demand function:

$Q_X = 5 - 3P_X - .5P_Y + 2\ P_Y + .03M$

Where:

P_Y = $10, the price of good Y

P_Z = $5, the price of good Z

M = $1000, the income of the buyer

$Q_X = 5 - 3P_X + .5(10) - 2(5) + .03(1000)$

$Q_X = 40 - 3P_X$

$P_X = 13.33 - .33Q_X$

If the P_X were $5, then the Q_X demanded is 25 units.

$$E_M = \frac{\Delta Q}{\Delta M} * \frac{M}{Q} = (+.03) * \frac{1000}{25}$$

$$= (.03) * 40 = +1.2$$

It is a "superior" good; $E_M > 1$

If $0 < E_M < 1$, it would be a normal good.

If $E_M < 0$, it is an inferior good

Income Elasticity - Examples

Selected E_M Examples	
Alcoholic Beverages	**0.29**
Restaurant Meals	**1.61**
Motion Pictures	**0.81**
Books and Maps	**1.67**
Dentists	**0.38**
Furniture	**2.60**
Funeral Expenses	**.48**
Auto Repair	**1.03**
Taxicabs	**1.14**
Toys	**.59**

Source: Hothakker and Taylor, 1970

These are estimated short run elasticities with respect to total expenditures rather than income. Savings is omitted from the analysis.

Restaurant Meals, Books, Furniture and Taxicabs are superior goods. Auto repair might be considered a superior good.

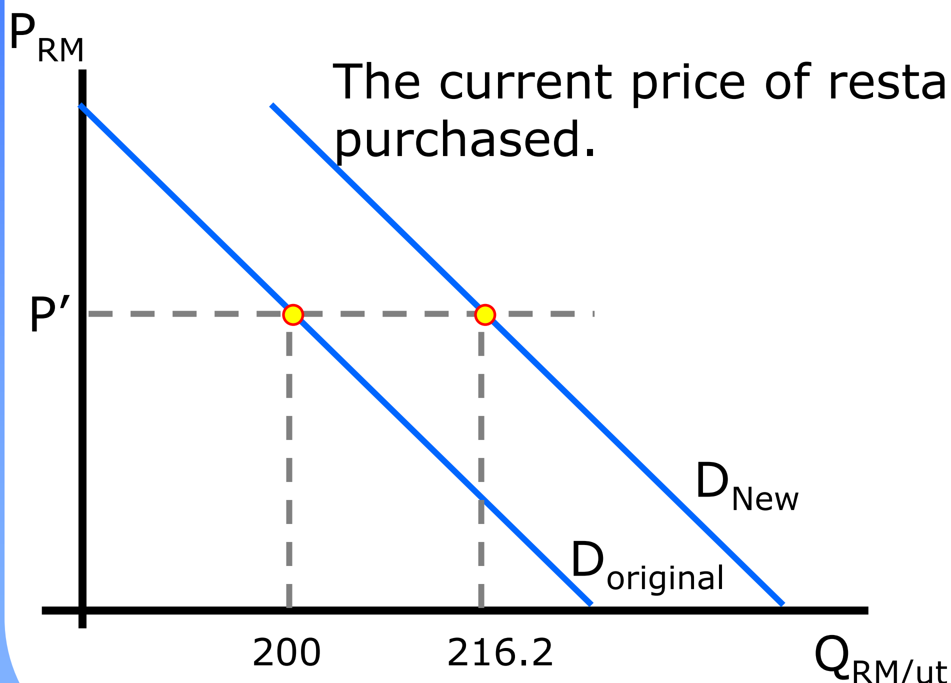
Alcoholic beverages, motion pictures, dentists, funeral expenses and toys are normal goods.

E_M is a measure of how much a change in income will shift the demand for the good.

Income Elasticity - Interpretation

Using restaurant meals as an example: $E_M = 1.61$.

If it was estimated that incomes would increase 5% over the next year, how much would the demand for restaurant meals increase?



The current price of restaurant meals is P’, 200 units are purchased.

$$E_M = \frac{\Delta Q}{\Delta M} * \frac{M}{Q} = +1.61$$

$$E_M = +1.61 = \frac{\%\Delta Q}{\%\Delta M}$$

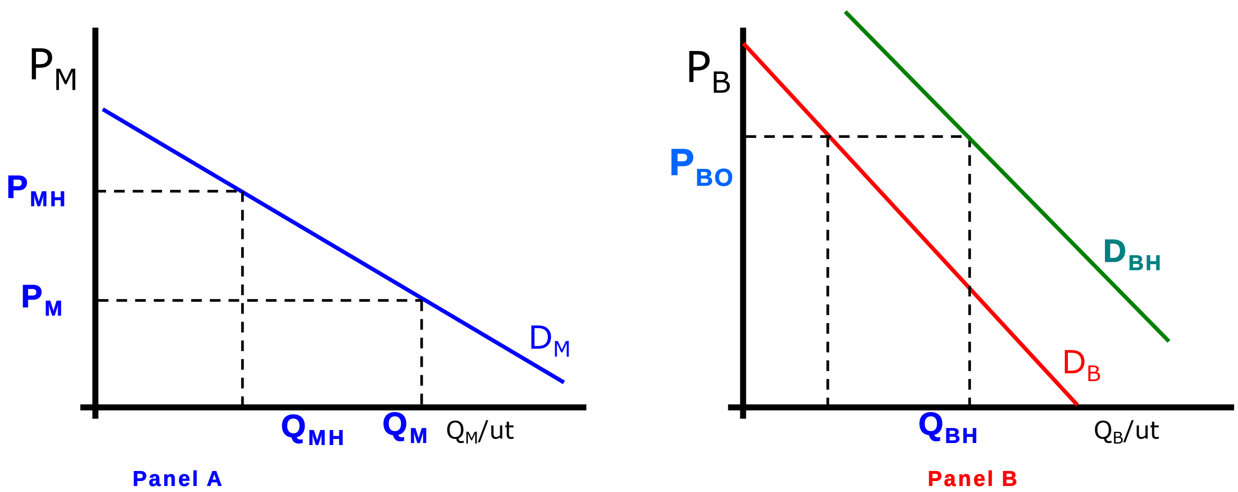
$$+1.61 = \frac{\%\Delta Q}{+5\%}$$

$$(+1.61)(5\%) = 8.1\% = \%\Delta Q$$

Cross Elasticity - E_{XY}

Cross elasticity is a measure of how a change in the price of one good changes the demand for another; measure degree of complementarity and substitutability.

$$E_{XY} \equiv \frac{\%\Delta Q_X}{\%\Delta P_Y} = \frac{\Delta Q_X}{\Delta P_Y} * \frac{P_Y}{Q_X}$$



In this example, the price of M increases to P_{MH}. As a result, the demand for good B increases to D_{BH}.

More of good B is purchased at the same price P_{BO}.

Cross Elasticity - E_{XY}

Cross elasticity has been used to identify markets in court cases, e.g. US v. ALCOA-Rome Cable Case (1964), Brown Shoe v. US Case (1962) and US v. Du Pont Cellophane Case (1956).

If $E_{XY} > 0$ this is evidence that goods X and Y are substitutes (not proof)

If $E_{XY} < 0$ this is evidence that goods X and Y are complements, (not proof)

E_{XY} is not the same as E_{YX},
Beef may be a better substitute for pork than pork is for beef.

Berndt, Friedlaender and Chaing (1990) estimate the EXY between Ford and GM at 7.01. A 1% increase in the price of Fords will result in a 7% increase in the demand for GM cars.