Carry out Problem 4.30 in three parts, as follows:

(a) Calculate the 2×2 matrix $\mathsf{S}_r \equiv \hat{r} \cdot \vec{\mathsf{S}}$, where $\hat{r}$ is given by Eq. [4.154], and $\vec{\mathsf{S}} \equiv \mathsf{S}_x \hat{\imath} + \mathsf{S}_y \hat{\jmath} + \mathsf{S}_z \hat{k}$, using the Pauli spin matrices in Eq. [4.148].

(b) Show that the two spinors $\chi_{\pm}^{(r)}$ in Eq. [4.155] are normalized, that they are eigenvectors of S_r, and determine their corresponding eigenvalues.

(c) The "direction" $\hat{n}$ of a normalized Pauli spinor χ can be defined as $\hat{n} \equiv \langle \chi | \vec{\mathsf{S}} | \chi \rangle / (\hbar/2)$. Calculate the directions of $\chi_{+}^{(r)}$ and $\chi_{-}^{(r)}$. How do these directions compare to $\hat{r}$ of Eq. [4.154]?