Instructions:
2. Work each problem on the exam booklet in the space provided.
3. Write neatly and clearly for partial credit. Cross out any material you do not want graded.

Name: ____________________________

Problem 1: ________________________/20
Problem 2: ________________________/20
Problem 3: ________________________/40
Problem 4: ________________________/20
Total: ____________________________/100

Singly-Excited Magnetic Structure: $W_m + W_m' = \lambda i$.  

\[
W_m = W_m(\lambda, x) = \int_{\lambda_0}^{\lambda} i(\lambda', x) d\lambda' \quad W'_m = W'_m(i, x) = \int_{i_0}^{i} \lambda(i', x) di'
\]

\[
F_e = F_e(\lambda, x) = -\frac{\partial W_m(\lambda, x)}{\partial x} \quad F_e = F_e(i, x) = \frac{\partial W'_m(i, x)}{\partial x}
\]

\[
i = i(\lambda, x) = \frac{\partial W_m(\lambda, x)}{\partial \lambda} \quad \lambda = \lambda(i, x) = \frac{\partial W'_m(i, x)}{\partial i}
\]

Doubly-Excited Magnetic Structure: $W_m + W_m' = \lambda_1 i_1 + \lambda_2 i_2$.  

\[
W_m = W_m(i_1, i_2, x) = \int_{i_1}^{i_1'} \lambda_1(i_1', 0, x) di_1' + \int_{i_2}^{i_2'} \lambda_2(i_1, i_2', x) di_2' 
\]

\[
F_e = F_e(i_1, i_2, x) = \frac{\partial W_m'}{\partial x}
\]
Problem 1 (20 Points)

Figure (b) is an equivalent circuit of the transformer circuit shown in Figure (a).

(a) Complete Figure (b) by adding the value of the load resistance referred to the primary side.

(b) Using either circuit or both, find \( \tilde{I}_1, \tilde{I}_2, \tilde{V}_2, \tilde{V}_1 \), and the real power \( P_L \) lost in the 5-ohm load resistor.

Alternate Solution:

\[
\tilde{I}_1 = \frac{100 \angle 0^\circ}{20 + j20} = \frac{100 \angle 0^\circ}{20\sqrt{2} \angle 45^\circ} = 2.5\sqrt{2} \angle -45^\circ \text{ Arms}
\]

\[
\tilde{I}_2 = 2\tilde{I}_1 = 5\sqrt{2} \angle -45^\circ \text{ Arms}
\]

\[
\tilde{V}_2 = 2\tilde{I}_2 = 25\sqrt{2} \angle -45^\circ \text{ Vrms}
\]

\[
\tilde{V}_1 = 2\tilde{V}_2 = 50\sqrt{2} \angle -45^\circ \text{ Vrms}
\]

\[
P_L = 5|\tilde{I}_2|^2 = 5 \times (5\sqrt{2})^2 = 250 \text{ W}
\]
Problem 2 (20 Points)

A 48-kVA, 2400/120-V, 60-Hz single-phase transformer is modeled using a series impedance model that assumes negligible core losses. Two short-circuit tests are performed on this transformer as shown below. Fill in the missing information in the two tables of expected results from these tests.

Solution:

\[ a = \frac{2400}{120} = 20 \]

\[ Z_{eH} = \frac{V_H}{I_H} = \frac{120}{20} = 6 \ \Omega \quad \Rightarrow \quad Z_{eL} = \frac{Z_{eH}}{a^2} = \frac{6}{20^2} = 15 \text{ m}\Omega \]

\[ R_{eH} = \frac{P_{sc}}{I_H^2} = \frac{672}{20^2} = 1.68 \ \Omega \quad \Rightarrow \quad R_{eL} = \frac{R_{eH}}{a^2} = \frac{1.68}{20^2} = 4.2 \text{ m}\Omega \]

\[ X_{eH} = \sqrt{Z_{eH}^2 - R_{eH}^2} = \sqrt{6^2 - 1.68^2} = 5.76 \ \Omega \quad \Rightarrow \quad X_{eL} = \frac{X_{eH}}{a^2} = \frac{5.76}{20^2} = 14.4 \text{ m}\Omega \]

\[ V_L = Z_{eL}I_L = 0.015 \times 400 = 6 \text{ V}, \quad P_{sc} = R_{eL}I_L^2 = 0.0042 \times 400^2 = 672 \text{ W} \]
The above electromechanical device has a movable part that is constrained to move only in the horizontal direction. The distances $x$ and $y$ defining the displacements of the movable part from the edges of the static part are constrained by $x + y = 2a$ where $2a$ is the sum of the two air gap lengths. Assume an infinitely-permeable core and neglect fringing and leakage effects.

(a) Specify each of the reluctances $R_x$ and $R_y$ shown in the equivalent magnetic circuit representation in terms of $\mu_o$, $x$, $y$, $w$ and $d$.

$$ R_x = \frac{1}{\mu_o w d} \frac{x}{\mu_o w d} = \frac{x}{\mu_o w d} $$

$$ R_y = \frac{1}{\mu_o 2w d} \frac{y}{2 \mu_o w d} $$

(b) Derive the flux-current relationship for this device in the form

$$ \lambda = N\phi = L(x,y)i $$

and specify $L(x,y)$ in terms of $\mu_o$, $w$, $d$, $N$, $x$, and $y$.

$$ R = R_x + R_y = \frac{2x + y}{2 \mu_o w d} $$

$$ \phi = \frac{Ni}{R} = \frac{2 \mu_o w d N i}{2x + y} $$

$$ \lambda = N\phi = \frac{2 \mu_o w d N^2 i}{2x + y} = L(x,y)i $$

$$ L(x,y) = \frac{2 \mu_o w d N^2}{2x + y} $$
(c) Substitute $y = 2a - x$ in Part (b) and derive the flux-current relationship in the form $\lambda = L(x)i$.

\[
\lambda = \frac{2\mu_0wdN^2i}{2x + (2a - x)} = \frac{2\mu_0wdN^2i}{x + 2a} = L(x)i
\]

\[
L(x) = \frac{2\mu_0wdN^2}{x + 2a}
\]

(d) Find the electromagnetic force $F_e = F_e(i, x)$.

\[
W_m' = W_m'(i, x) = \frac{1}{2}L(x)i^2 = \frac{\mu_0wdN^2i^2}{x + 2a}
\]

\[
F_e = F_e(i, x) = \frac{\partial W_m'}{\partial x} = \frac{1}{2}i^2 \frac{dL}{dx} = -\frac{\mu_0wdN^2i^2}{(x + 2a)^2}
\]

(e) Substitute $x = 2a - y$ in Part (b) and derive the flux-current relationship in the form $\lambda = L(y)i$.

\[
\lambda = \frac{2\mu_0wdN^2i}{2(2a - y) + y} = \frac{2\mu_0wdN^2i}{4a - y} = L(y)i
\]

\[
L(y) = \frac{2\mu_0wdN^2}{4a - y}
\]

(f) Find the electromagnetic force $F_e = F_e(i, y)$.

\[
W_m' = W_m'(i, y) = \frac{1}{2}L(y)i^2 = \frac{\mu_0wdN^2i^2}{4a - y}
\]

\[
F_e = F_e(i, y) = \frac{\partial W_m'}{\partial y} = \frac{1}{2}i^2 \frac{dL}{dy} = \frac{\mu_0wdN^2i^2}{(4a - y)^2}
\]

(g) The N-turn coil is excited with a DC current $I$ and the horizontal slab is initially at rest at $x = y = a$. If released, will the slab move to the left or to the right? (Explain.) **Answer**: To the left.

\[
F_e = F_e(i = I, x = a) = -\frac{\mu_0wdN^2I^2}{(a + 2a)^2} = -\frac{\mu_0wdN^2I^2}{9a^2} < 0 \text{ in } x\text{-direction}
\]

\[
F_e = F_e(i = I, y = a) = \frac{\mu_0wdN^2I^2}{(4a - a)^2} = \frac{\mu_0wdN^2I^2}{9a^2} > 0 \text{ in } y\text{-direction}
\]

(h) If the direction of the current $I$ is reversed in the coil, will the slab move to the left or to the right from rest at $x = y = a$? (Explain.) **Answer**: To the left. The electromagnetic force is independent of the direction of the current since $F_e$ is proportional to $i^2$. 

\[
W'_m = \frac{1}{2}L(x)i^2 = \frac{\mu_0wdN^2i^2}{x + 2a}
\]

\[
F_e = \frac{\partial W'_m}{\partial x} = \frac{1}{2}i^2 \frac{dL}{dx} = -\frac{\mu_0wdN^2i^2}{(x + 2a)^2}
\]
**Problem 4** (20 Points)

A rotating two-pole machine has two windings represented by the following flux-current relationships:

\[
\begin{align*}
\lambda_1 &= 0.15i_1 + (0.08 \cos \theta)i_2 \\
\lambda_2 &= (0.08 \cos \theta)i_1 + 0.06i_2
\end{align*}
\]

(a) Is this system electrically linear?  Yes  No.

(b) Find the magnetic coenergy \( W'_m = W'_m(i_1, i_2, \theta) \).

\[
\begin{align*}
W'_m &= W'_m(i_1, i_2, \theta) = \frac{1}{2}\lambda_1i_1 + \frac{1}{2}\lambda_2i_2 \\
&= \frac{1}{2}(0.15i_1 + 0.08i_2 \cos \theta)i_1 + \frac{1}{2}((0.08 \cos \theta)i_1 + 0.06i_2)i_2 \\
&= 0.075i_1^2 + 0.08i_1i_2 \cos \theta + 0.03i_2^2
\end{align*}
\]

(c) Find the electromagnetic torque \( T_e \) at \( \theta = \pi/2 \) rad and \( i_1 = i_2 = 5 \) A.

\[
\begin{align*}
T_e &= T_e(i_1, i_2, \theta) = \frac{\partial W'_m}{\partial \theta} = -0.08i_1i_2 \sin \theta \\
T_e &= T_e\left(5, 5, \frac{\pi}{2}\right) = -0.08 \times 5 \times 5 \times \sin \frac{\pi}{2} = -2 \text{ N-m}
\end{align*}
\]

(d) Find the magnetic energy \( W_m \) stored in the coupling field at \( \theta = \pi/2 \) rad and \( i_1 = i_2 = 5 \) A.

\[
\begin{align*}
W_m &= W'_m = W'_m\left(5, 5, \frac{\pi}{2}\right) = 0.075 \times 5^2 + 0.08 \times 5 \times 5 \times \cos \frac{\pi}{2} + 0.03 \times 5^2 = 2.625 \text{ J}
\end{align*}
\]