Instructions:

2. Work each problem on the exam booklet in the space provided.
3. Write neatly and clearly for partial credit. Cross out any material you do not want graded.

Name: __________________________

Problem 1: ________________________/20
Problem 2: ________________________/30
Problem 3: ________________________/25
Problem 4: ________________________/25
Total: ___________________________/100

Singly-Excited Magnetic Structure: $W_m + W'_m = \lambda i$.

$W_m = W_m(\lambda, x) = \int_0^\lambda i(\lambda', x) \, d\lambda'$
$W'_m = W'_m(i, x) = \int_0^i \lambda(i', x) \, di'$

$F_e = F_e(\lambda, x) = -\frac{\partial W_m(\lambda, x)}{\partial x}$
$F_e = F_e(i, x) = \frac{\partial W'_m(i, x)}{\partial x}$

$i = i(\lambda, x) = \frac{\partial W_m(\lambda, x)}{\partial \lambda}$
$\lambda = \lambda(i, x) = \frac{\partial W'_m(i, x)}{\partial i}$

Doubly-Excited Magnetic Structure: $W_m + W'_m = \lambda_1 i_1 + \lambda_2 i_2$.

$W_m = W_m(i_1, i_2, x) = \int_0^{i_1} \lambda_1(i_1', 0, x) \, di_1' + \int_0^{i_2} \lambda_2(i_1, i_2', x) \, di_2'$

$F_e = F_e(i_1, i_2, x) = \frac{\partial W'_m}{\partial x}$
Problem 1 (20 Points)

A 48-kVA, 2400/120-V, 60-Hz single-phase transformer is modeled by a series impedance and assumes the core losses are negligible. A sloppy engineer-in-training records the following results of a short-circuit test:

\[
V = 120 \text{ V} \ll 2400 \text{ V} \implies V_H = 120 \text{ V} \\
I = 20 \text{ A} = \frac{48,000}{2400} = I_{H,\text{rated}} \implies I_H = 20 \text{ A} \\
P = 672 \text{ W}
\]

(You may assume that rated currents flow in the primary and secondary windings during the short-circuit test.)

(a) Check the correct answer below:

(i) The primary winding is shorted during this test.
(ii) The secondary winding is shorted during this test.

\[
\begin{align*}
|I_H| &= 20 \text{ A} \\
R_{eH} &= 1.68 \Omega \\
X_{eH} &= j5.76 \Omega \\
V_H &= 120 \text{ V} \\
L_{H} &= R_{eH} I_H^2 = 672 / 20^2 = 1.68 \Omega \\
V_L &= 0 \text{ V}
\end{align*}
\]

(b) Find \(R_{eH}\) and \(X_{eH}\) where the subscript \(H\) refers to the high-voltage side.

\[
\begin{align*}
P_{sc} &= R_{eH} I_H^2 \implies R_{eH} = \frac{P_{sc}}{I_H^2} = \frac{672}{20^2} = 1.68 \Omega \\
Z_{eH} &= \frac{V_H}{I_H} = \frac{120}{20} = 6 \Omega = \sqrt{R_{eH}^2 + X_{eH}^2} \\
X_{eH} &= \sqrt{Z_{eH}^2 - R_{eH}^2} = \sqrt{6^2 - 1.68^2} = 5.76 \Omega \\
or \quad S_{sc} &= V_H I_H = 120 \times 20 = 2400 \text{ VA} = \sqrt{P_{sc}^2 + Q_{sc}^2} \\
Q_{sc} &= \sqrt{S_{sc}^2 - P_{sc}^2} = \sqrt{2400^2 - 672^2} = 2304 \text{ VAr} \\
Q_{sc} &= X_{eH} I_H^2 \implies X_{eH} = \frac{Q_{sc}}{I_H^2} = \frac{2304}{20^2} = 5.76 \Omega
\end{align*}
\]
Problem 2 (30 Points)

A 1-kVA, 60-Hz, 400/100-V transformer delivers rated load at rated secondary voltage (low-voltage side) and 0.8 pf lagging. This transformer has the following parameters:

<table>
<thead>
<tr>
<th>$R_1$ (Ω)</th>
<th>$X_1$ (Ω)</th>
<th>$R_2$ (Ω)</th>
<th>$X_2$ (Ω)</th>
<th>$R_{e1}$ (Ω)</th>
<th>$X_{m1}$ (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80</td>
<td>8.0</td>
<td>0.050</td>
<td>0.50</td>
<td>8000</td>
<td>8000</td>
</tr>
</tbody>
</table>

(a) Find $R_{e1}$ and $X_{e1}$.

\[
R_{e1} = R_1 + a^2 R_2 = 0.80 + 4^2 \times 0.050 = 1.6 \, \Omega \\
X_{e1} = X_1 + a^2 X_2 = 8.0 + 4^2 \times 0.50 = 16 \, \Omega
\]

(b) Find $\tilde{S}_2$, $\tilde{V}_2$, $\tilde{V}_2'$, $\tilde{I}_2$, $\tilde{I}_2'$, and $\tilde{V}_1$.

\[
\tilde{S}_2 = 1000 \angle \cos^{-1} 0.8 = 1000 \angle 36.87^\circ \, \text{VA} = 800 + j600 \, \text{VA} \\
\tilde{V}_2 = 100 \angle 0^\circ \, \text{V} \\
\tilde{V}_2' = a\tilde{V}_2 = 4 \times 100 \angle 0^\circ = 400 \angle 0^\circ \, \text{V} \\
\tilde{S}_2 = \tilde{V}_2 \tilde{I}_2^* \implies \tilde{I}_2 = \frac{\tilde{S}_2}{\tilde{V}_2^*} = \frac{1000 \angle -36.87^\circ}{100} = 10 \angle -36.87^\circ \, \text{A} \\
\tilde{I}_2' = \frac{1}{a} \tilde{I}_2 = \frac{1}{4} \times 10 \angle -36.87^\circ = 2.5 \angle -36.87^\circ = 2.0 - j1.5 \, \text{A} \\
\tilde{V}_1 = (R_{e1} + jX_{e1})\tilde{I}_2 + \tilde{V}_2' \\
= (1.6 + j16)(2.0 - j1.5) + (400 + j0) \\
= 427.2 + j29.6 \approx 428 \angle 3.96^\circ \, \text{V}
\]

(c) Compute the efficiency $\eta$ of this transformer in percent.

\[
\eta = \frac{P_2}{P_2 + R_{e1}|\tilde{I}_2'|^2 + |\tilde{V}_1|^2/R_{e1}} \times 100\% = \frac{800}{800 + 1.6 \times 2.5^2 + 428^2/8000} \times 100\% \approx 96.1\%
\]
Problem 3 (25 Points)

A U-shaped electromagnet is used to lift an iron bar at a distance \( x = 1 \) mm. The cross-sectional area of each pole face is \( A = wd = 12\pi \) cm\(^2\). (Assume that the permeability of the magnetic materials is infinite and neglect fringing and leakage effects.)

(a) Express the inductance \( L = L(x) \) of this magnetic structure in terms of \( \mu_0, A, N, \) and \( x \).

\[
\phi = \frac{Ni}{R_e} = \frac{Ni}{2R_x} = \left(\frac{N}{2R_x}\right)i
\]

\[
\lambda = N\phi = \left(\frac{N^2}{R_e}\right)i = \left(\frac{N^2}{2R_x}\right)i = L(x)i
\]

\[
L(x) = \frac{N^2}{R_e} = \frac{N^2}{2R(x)} = \frac{\mu_0AN^2}{2x}
\]

(b) Show that the magnetic force \( F_e = F_e(\lambda, x) \) can be expressed as

\[
F_e = F_e(\lambda, x) = -\frac{\lambda^2}{\mu_0AN^2}
\]

\[
\lambda = L(x)i = \frac{\mu_0AN^2i}{2x}
\]

\[
W_m = W_m(\lambda, x) = \frac{1}{2}\lambda i = \frac{\lambda^2}{2L(x)} = \frac{\lambda^2}{2} \times \frac{2x}{\mu_0AN^2} = \frac{x\lambda}{\mu_0AN^2}
\]

\[
F_e = F_e(\lambda, x) = -\frac{\partial W_m}{\partial x} = -\frac{\lambda^2}{\mu_0AN^2}
\]

(b) Compute \( |F_e| \) (N) numerically if the magnetic flux density \( B_a = 0.8 \) T in the air gap.

\[
|F_e| = \frac{\lambda^2}{\mu_0AN^2} = \frac{(NB_aA)^2}{\mu_0AN^2} = \frac{B_a^2A}{\mu_0} = \frac{0.8^2 \times 12\pi \times 10^{-4}}{4\pi \times 10^{-7}} = 1920 \text{ N}
\]
Problem 4 (25 Points)

A doubly-excited rotating machine has one stator winding and one rotor winding characterized by the following flux-current relationships:

\[
\lambda_s = (0.75 + 0.25 \cos 2\theta) i_s + (0.8 \cos \theta) i_r \\
\lambda_r = (0.8 \cos \theta) i_s + (0.45 + 0.15 \cos 2\theta) i_r
\]

(a) Is this system electrically linear? **Yes** No (Underline the correct answer.)

(b) Find the magnetic coenergy of the system \( W'_m = W'_m(i_s, i_r, \theta) \).

\[
W'_m(i_s, i_r, \theta) = \frac{1}{2} \lambda_s i_s + \frac{1}{2} \lambda_r i_r
\]

\[
= \frac{1}{2} [(0.75 + 0.25 \cos 2\theta) i_s + (0.8 \cos \theta) i_r] i_s + \frac{1}{2} [(0.8 \cos \theta) i_s + (0.45 + 0.15 \cos 2\theta) i_r] i_r
\]

\[
= (0.75 + 0.25 \cos 2\theta) \times \frac{i_s^2}{2} + (0.4 \cos \theta) i_s i_r + (0.4 \cos \theta) i_s i_r + (0.45 + 0.15 \cos 2\theta) \times \frac{i_r^2}{2}
\]

or

\[
W'_m(i_s, i_r, \theta) = \int_0^{i_s} \lambda_1(i'_s, 0, \theta) \, di'_s + \int_0^{i_r} \lambda_2(i_s, i'_r, \theta) \, di'_r
\]

\[
= \int_0^{i_s} [(0.75 + 0.25 \cos 2\theta) i'_s + (0.8 \cos \theta)(0)] \, di'_s
\]

\[
+ \int_0^{i_r} [(0.8 \cos \theta) i_s + (0.45 + 0.15 \cos 2\theta) i'_r] \, di'_r
\]

\[
= (0.75 + 0.25 \cos 2\theta) \times \frac{i_s^2}{2} + (0.8 \cos \theta) i_s i_r + (0.45 + 0.15 \cos 2\theta) \times \frac{i_r^2}{2}
\]

(c) Find the electromagnetic torque \( T_e = T_e(i_s, i_r, \theta) \).

\[
T_e = T_e(i_s, i_r, \theta) = \frac{\partial W'_m}{\partial \theta}
\]

\[
= -0.25 i_s^2 \sin 2\theta - 0.8 i_s i_r \sin \theta - 0.15 i_r^2 \sin 2\theta
\]

(d) Find the electromagnetic torque value for \( i_s = i_r = 1 \) A and \( \theta = 90^\circ \).

\[
T_e = 0.25 i_s^2 \sin 2\theta - 0.8 i_s i_r \sin \theta - 0.15 i_r^2 \sin 2\theta
\]

\[
= -(0.25)(1)^2 \sin 180^\circ - (0.8)(1)(1) \sin 90^\circ - (0.15)(1)^2 \sin 180^\circ
\]

\[
= -0.8 \text{ N-m}
\]