Instructions:

2. Work each problem on the exam booklet in the space provided.
3. Write neatly and clearly for partial credit. Cross out any material you do not want graded.

Name: ________________________________

Problem 1: _______________________/25
Problem 2: _______________________/25
Problem 3: _______________________/20
Problem 4: _______________________/30
Total: _______________________/100

Useful Equations:

\[ P_{IN,3ph} = 3P_{IN,1ph} = \frac{3V_a E_a}{X_s} \sin(-\delta) = P_e = P_m = P_{out} + P_{rot} \]
\[ Q_{IN,3ph} = 3Q_{IN,1ph} = \frac{3V_a^2}{X_s} - \frac{3V_a E_a}{X_s} \cos(-\delta) \]

\[ E_a = E_a / \delta \]
\[ V_a = V_a \angle \delta \]
Problem 1 (25 Points)

(a) Write four independent equations (three Ampere’s laws and one Gauss’ law) in $H_1$, $H_2$, $H_3$, and $H_4$.

(b) Solve the above equations for $H_1$, $H_2$, $H_3$, and $H_4$, and plot $H_s$ as a function of $\alpha$ in the air gap. Alternatively, you can use superposition and plot $H_s$ as a function of $\alpha$ in the air gap and clearly show the H-fields of both full-pitch coils 1-1' and 2-2' if each coil has $N_s/2$ turns.

(c) Using superposition, write down the fundamental component of the Fourier series expansion of $H_s(\alpha)$. (Do not simplify the two expressions using trigonometric identities.)
Problem 2 (25 Points)

A six-pole single-phase induction machine has one stator winding (sa-sa’) and two rotor windings (ra-ra’ and rb-rb’).

(a) By inspection of the various magnetic axes of this machine, fill in the missing inductances below:

\[
\begin{bmatrix}
\lambda_{sa} \\
\lambda_{ra} \\
\lambda_{rb}
\end{bmatrix} =
\begin{bmatrix}
L_{ls} + L_{ms} & M_{sr} \cos \theta & 0 \\
0 & L_{lr} + L_{mr} & 0 \\
0 & 0 & L_{lr} + L_{mr}
\end{bmatrix}
\begin{bmatrix}
i_{sa} \\
i_{ra} \\
i_{rb}
\end{bmatrix}
\]

(b) Find the magnetic coenergy \( W_m' = W_m'(i_{sa}, i_{ra}, i_{rb}, \theta) \).

(c) Find the magnetic torque \( T_e = T_e(i_{sa}, i_{ra}, i_{rb}, \theta) \) developed by this machine.
Problem 3 (20 Points)

Starting with

\[ \bar{S}_{IN,3ph} = 3\bar{V}_a\bar{I}_a \]

show that:

\[ P_{IN,3ph} = \frac{3V_a E_a}{X_s} \sin(-\delta) \]

\[ Q_{IN,3ph} = \frac{3V_a^2}{X_s} - \frac{3V_a E_a}{X_s} \cos(-\delta) \]
Problem 4 (30 Points)

\[
\begin{align*}
& \text{A 1-hp, 208-V, 60-Hz, three-phase, four-pole, round-rotor synchronous machine has the following parameters: } R_s \approx 0 \, \Omega, X_s = 28.8 \, \Omega, K_f = 0.3 \, \text{H. The machine is operated from a 208-V (line-to-line) three-phase source with a field current } I_f = 1 \, \text{A. Neglect rotational (windage and friction) and core losses.} \\
& \text{(a) Find the synchronous mechanical speed } \omega_{ms} \text{ (mech. rad/s).} \\
& \text{(b) Find the rated output mechanical power } P_m = P_{out} \text{ (W).} \\
& \text{(c) Find the rated output mechanical torque } T_m = T_{out} \text{ (N-m).}
\end{align*}
\]

\[P_{in,3\phi} = \frac{3V_aE_a}{X_s} \sin(-\delta)\]

\[Q_{in,3\phi} = \frac{3V_a^2}{X_s} - \frac{3V_aE_a}{X_s} \cos(-\delta)\]
(d) Assuming an efficiency $\eta = 74.6\%$, find the input real power $P_{in,3\text{ph}}$ (W).

(e) Find the generated voltage $E_a$ (V).

(f) Find the torque angle $\delta$ (deg).

(g) Find the input reactive power $Q_{in,3\text{ph}}$ (VAr).

(h) Find the input apparent power $S_{in,3\text{ph}}$ (VA).

(i) Find the stator current magnitude $I_a$ (A).

(j) Find the input power factor $\text{pf} = \cos \phi$. (Specify leading/lagging.)