1 Objective

The objectives of this experiment are to further understand machine dynamics through the mathematical simulation of synchronous machine step loading.

2 Background

The instantaneous rotor position of a synchronous machine may be written in mechanical radians as,

$$\theta_m(t) = \int_0^t \omega_m(t) \, dt + \theta_m(0)$$

(1)

where $\omega_m(t)$ is the instantaneous rotor speed in mechanical radians per second. For a 6-pole 60-Hz synchronous machine running at 1200 rpm, the steady-state shaft speed is equal to the mechanical synchronous speed

$$\omega_m(t) = \omega_{ms} = \left( \frac{2}{p} \right) \omega_s = \left( \frac{2}{6} \right) (2\pi 60)$$

(2)

and

$$\theta_m(t) = 40\pi t + \theta_m(0)$$

(3)

Defining $\theta(t)$ in electrical radians by

$$\theta(t) = \left( \frac{p}{2} \right) \theta_m(t)$$

(4)

and $\omega(t)$ in electrical radians per second by

$$\omega(t) = \left( \frac{p}{2} \right) \omega_m(t)$$

(5)

In steady state,

$$\omega(t) = \omega_s = 2\pi f_s = 2\pi 60$$

(6)

$$\theta(t) = 2\pi 60 t + \theta(0)$$

(7)

where $\theta(0) = (p/2) \theta_m(0)$. For a fixed voltage source, an electrical angular displacement may be written as

$$v_a(t) = \sqrt{2} V_a \cos(\theta_s(t)) = \sqrt{2} V_a \cos(\omega_s t + \theta_v)$$

(8)

where for a 60-Hz source, $\omega_s = 2\pi 60$ electrical radians per second. Define the power (torque) angle delta,

$$\delta(t) \triangleq \theta(t) - \theta_s(t) + \frac{\pi}{2}$$

(9)
where, $\theta(t)$, $\theta_s(t)$ and thus $\delta(t)$ are all in electrical radians. Newton’s second law for rotating bodies requires that

$$J \frac{d^2 \theta_m}{dt^2} = \sum T_+$$

where $\sum T_+$ is the sum of torques acting in the positive $+\theta_m$-direction. Since

$$\theta_m = \left( \frac{2}{p} \right) \theta$$

then

$$\frac{d^2 \theta_m}{dt^2} = \left( \frac{2}{p} \right) \frac{d^2 \theta}{dt^2}$$

Also note that

$$\frac{d^2 \theta}{dt^2} = \frac{d}{dt} \left( \frac{d\theta}{dt} + \omega_s \right) = \frac{d\delta}{dt}$$

and

$$\frac{d^2 \theta}{dt^2} = \frac{d}{dt} \left( \frac{d\theta}{dt} \right) = \frac{d\omega}{dt}$$

It can be shown that the steady-state electrical torque into a balanced $3\phi$ round-rotor $p$-pole synchronous motor may be expressed as

$$T_e = \frac{P_e}{\omega_{ms}} = \frac{P_e}{(2/p)\omega_s} = \frac{3|\tilde{V}_a||\tilde{E}_a|}{(2/p)\omega_sX_s} \sin(-\delta)$$

where $|\tilde{V}_a|$ is the line-to-neutral rms stator phase-a voltage, $X_s$ is the synchronous reactance, and

$$|\tilde{E}_a| = \frac{\omega_sM_{sf}I_f}{\sqrt{2}}$$

where $\omega_s$ is in electrical radians per second, and $\delta$ is in electrical radians. The damper (amortisseur) windings of a synchronous machine provide an asynchronous torque through induction which contributes only when $\omega_m \neq \omega_{ms}$ so that

$$T_D = D(\omega_m - \omega_{ms}) = D \left( \frac{2}{p} \right) (\omega - \omega_s)$$

The steady-state mechanical shaft torque is

$$T_m = \frac{P_m}{\omega_{ms}} = \frac{P_m}{(2/p)\omega_s}$$

where $P_m$ is the shaft load in watts.

Assuming that the steady-state electrical and mechanical torque expressions are valid for the relatively slow mechanical dynamics being investigated in this experiment, the equations which describe the synchronous machine dynamics are as follows:

$$\frac{d\delta}{dt} = \omega - \omega_s \quad \text{(D.E.1)}$$

$$\frac{d\omega}{dt} = \frac{1}{J} \left( \frac{p}{2} \right) (T_e - T_D - T_m) \quad \text{(D.E.2)}$$
2.1 The Problem

Consider a balanced 3φ 6-pole 60-Hz round-rotor synchronous motor with

\[ R_a = 0 \, \Omega \quad |V_a| = \frac{230}{\sqrt{3}} \, V \]

\[ X_s = 9.0 \, \Omega \quad D = 2 \, N\cdot m/s \]

\[ M_{sf} = 0.4 \, H \quad p = 6 \text{ poles/phase} \]

\[ I_f = 1.2 \, A \quad J = 0.4 \, \text{kg}\cdot m^2 \]

Let \( P_m = 0 \) at \( t = 0 \) so that the machine is unloaded, with \( \delta(0) = 0 \) and, in steady state, \( \omega(0) = \omega_s \). The maximum load power \( P_{m,max} \) (W) that the motor could handle theoretically is found by setting \( \delta = -90^\circ \) in Equation (15) and solving for power instead of torque:

\[ P_{m,max} = \frac{3|V_a||E_a|}{X_s} = \frac{3 \times (230/\sqrt{3}) \times (120\pi \times 0.4 \times 1.2/\sqrt{2})}{9} \approx 5664 \, W \]

a) Determine the stability of the synchronous motor (stable or unstable) by plotting the dynamic responses \( \delta(t) \) and \( \omega(t) \) to sudden shaft loads of \( P_m = 3000, 4000, 5000 \) Watts, each applied at \( t = 1.0 \) s using a “switch” block. Perform these simulations for \( D = 2 \) N-m and \( D = 0 \) N-m and comment on the machine stability in each case by filling out the following table:

<table>
<thead>
<tr>
<th>( P_m ) (W)</th>
<th>3000</th>
<th>4000</th>
<th>5000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D = 2 ) (N-m)</td>
<td>Stable/Unstable</td>
<td>Stable/Unstable</td>
<td>Stable/Unstable</td>
</tr>
<tr>
<td>( D = 0 ) (N-m)</td>
<td>Stable/Unstable</td>
<td>Stable/Unstable</td>
<td>Stable/Unstable</td>
</tr>
</tbody>
</table>

b) Use your Simulink model to find the critical \( P_m = P_{cr} \) and \( T_m = T_{cr} \) which can be added suddenly from no load and such that the machine will not lose synchronism with the network for \( D = 2 \) N-m. Repeat this simulation for the worst-case scenario of \( D = 0 \) N-m.

c) Verify that the critical \( T_{cr} \) satisfies the equal-area criterion discussed in class for the case of zero damping. In other words, verify numerically (using MATLAB) that the steady-state angle \( |\delta_{cr}| \) corresponding to \( T_m = T_{cr} \) (or \( P_m = P_{cr} \)) satisfies the criterion that the decelerating area \( A_1 \) equals the accelerating area \( A_2 \) when \( D = 0 \).

d) Submit a copy of your Simulink model and three plots of \( \delta(t) \) for \( P_m = 3000, 4000, 5000 \) Watts when \( = 0 \) on separate graphs. Add comments for each graph.