Instructions:
2. Work each problem on the exam booklet in the space provided.
3. Write neatly and clearly for partial credit. Cross out any material you do not want graded.

Name: ________________________________

Problem 1: __________________________/20
Problem 2: ___________________________/30
Problem 3: ___________________________/30
Problem 4: ___________________________/20
Total: _______________________________/100

Singly-Excited Magnetic Structure: \( W_m + W'_m = \lambda i. \)

\[ W_m = W_m(\lambda, x) = \int_0^\lambda i(\lambda', x) \, d\lambda' \quad W'_m = W'_m(i, x) = \int_0^i \lambda(i', x) \, di' \]
\[ F_e = F_e(\lambda, x) = -\frac{\partial W_m(\lambda, x)}{\partial x} \quad F_e = F_e(i, x) = \frac{\partial W'_m(i, x)}{\partial x} \]
\[ i = i(\lambda, x) = \frac{\partial W_m(\lambda, x)}{\partial \lambda} \quad \lambda = \lambda(i, x) = \frac{\partial W'_m(i, x)}{\partial i} \]

Doubly-Excited Magnetic Structure: \( W_m + W'_m = \lambda_1 i_1 + \lambda_2 i_2. \)

\[ W'_m = W'_m(i_1, i_2, x) = \int_0^{i_1} \lambda_1(i_1', 0, x) \, di'_1 + \int_0^{i_2} \lambda_2(i_1, i_2', x) \, di'_2 \]
\[ F_e = F_e(i_1, i_2, x) = \frac{\partial W'_m}{\partial x} \]
Problem 1 (20 Points)

(a) Refer the voltage source and the reactances to the side of the 24-ohm load resistor and fill in their values in the impedance diagram below:

(b) Compute the load current phasor \( \tilde{I}_L \) (A) in polar form.
\[
\tilde{I}_L = \frac{200 \angle 0^\circ}{24 + j7} = \frac{200 \angle 0^\circ}{25 \angle 16.26^\circ} = 8 \angle -16.26^\circ \text{ A}
\]

(c) Calculate the real power \( P_L \) (W) dissipated in the 24-ohm resistor.
\[
P_L = R_L |\tilde{I}_L|^2 = 24 \times 8^2 = 1536 \text{ W}
\]
**Problem 2 (30 Points)**

A 3-kVA, 240-V/120-V, 60-Hz two-winding transformer yields the following test data:

- Open-Circuit Test: 240 V, 0.625 A, 120-W
- Short-Circuit Test: 12.5 V, 12.5 A, 43.75 W

(a) Determine the equivalent parameters ($R_{e1}$, $X_{e1}$, $R_{c1}$, and $X_{m1}$) referred to the primary side of this transformer.

![Diagram of transformer](image)

\[
\begin{align*}
P_{oc} &= \frac{V_1^2}{R_{c1}} \implies R_{c1} = \frac{V_1^2}{P_{oc}} = \frac{240^2}{120} = 480 \Omega \\
S_{oc} &= V_1I_1 = 150 \text{ VA} \implies Q_{oc} = \sqrt{S_{oc}^2 - P_{oc}^2} = \sqrt{150^2 - 10^2} = 90 \text{ VAr} \\
Q_{oc} &= \frac{V_1^2}{X_{m1}} \implies X_{m1} = \frac{V_1^2}{Q_{oc}} = \frac{240^2}{90} = 640 \Omega \\
P_{sc} &= R_{e1}I_1^2 \implies R_{e1} = \frac{P_{sc}}{I_1^2} = \frac{43.75}{12.5^2} = 0.28 \Omega \\
S_{sc} &= V_1I_1 = 156.25 \text{ VA} \implies Q_{sc} = \sqrt{S_{sc}^2 - P_{sc}^2} = \sqrt{156.25^2 - 43.75^2} = 150 \text{ VAr} \\
Q_{sc} &= X_{e1}I_1^2 \implies X_{e1} = \frac{Q_{sc}}{I_1^2} = \frac{150}{12.5^2} = 0.96 \Omega
\end{align*}
\]

(b) Determine the primary voltage $\tilde{V}_1$ (V) of the transformer when it is delivering a load of 2.4 kVA at unity power factor and at rated secondary voltage.

\[
\begin{align*}
\tilde{I}_2 &= \frac{2400}{120} \angle 0^\circ = 20 \angle 0^\circ \text{ A} \implies \tilde{I}_2' = 10 \angle 0^\circ = 10 + j0 \text{ A} \\
\tilde{V}_2 &= 120 \angle 0^\circ \text{ V} \implies \tilde{V}_2' = 240 \angle 0^\circ = 240 + j0 \text{ V} \\
\tilde{V}_1 &= (R_{e1} + jX_{e1})\tilde{I}_2' + \tilde{V}_2' = (0.28 + j0.96)(10 + j0) + 240 = 242.8 + j9.6 \approx 243 \angle 2.26^\circ \text{ V}
\end{align*}
\]

(c) Determine the efficiency and regulation of this transformer under the conditions of Part (b).

\[
\begin{align*}
\eta &= \frac{P_2}{P_2 + R_{e1}I_2^2 + \frac{V_1^2}{R_{c1}}} = \frac{2400 \times 0.8}{2400 \times 0.8 + 0.28 \times 10^2 + 243^2/480} \approx 0.941 = 94.1\% \\
R &= \frac{V_1 - V_1'}{V_2'} = \frac{243 - 240}{240} \approx 0.0125 = 1.25\%
\end{align*}
\]
Problem 3 (30 Points)

The movable part of the above electromagnet is kept in equilibrium by a spring force. Assuming that the permeability of the magnetic materials is infinite:

(a) Finish drawing the magnetic circuit representation of this magnetic structure by adding polarity marks to the magnetomotive force (mmf) $F = Ni$ and specifying the value of the effective reluctance $R_e$ seen by the mmf in terms of $\mu_o$, $w$, $d$, and $x$.

$$R_e = \frac{1}{\mu_o wd}$$

$$R_x = \frac{1}{2} R_x$$

$$R_e = R_x + R_x \parallel R_x = \frac{3}{2} R_x = \frac{3x}{2\mu_o wd}$$

(b) Deduce the inductance $L = L(x)$ of this coil in terms of $\mu_o$, $w$, $d$, $N$, and $x$.

$$L = \frac{N^2}{R_e} = \frac{2N^2}{3R_x} = \frac{2\mu_o wdN^2}{3x}$$

(c) Find the electromagnetic coenergy $W'_m = W'_m(i, x)$ of this system in terms of $\mu_o$, $w$, $d$, $N$, $x$, and $i$.

$$W'_m = W'_m(i, x) = \frac{1}{2} Li = \frac{1}{2} L(x)i^2 = \frac{\mu_o wdN^2 i^2}{3x}$$

(d) Find the electromagnetic force $F_e = F_e(i, x)$ in terms of $\mu_o$, $w$, $d$, $N$, $x$, and $i$.

$$F_e = \frac{dW'_m}{dx} = -\frac{\mu_o wdN^2 i^2}{3x^2}$$

(e) If $i(t) = \sqrt{2}I_{rms} \sin \omega t$, $w = d = 5 \text{ cm}$, $x = 2 \text{ mm}$, $N = 750$ turns, and $F_{spring} = 30 \text{ N}$, calculate the needed rms value $I_{rms}$ of the current $i(t)$.

$$|F_e| = \frac{\mu_o wdN^2 i^2}{3x^2} = \frac{\mu_o wdN^2}{3x^2} \left(\sqrt{2}I_{rms} \sin \omega t\right)^2$$

$$= \frac{\mu_o wdN^2}{3x^2} \left(2I_{rms} \sin^2 \omega t\right) = \frac{\mu_o wdN^2}{3x^2} \times I_{rms}^2 (1 - \cos 2\omega t)$$

$$|F_{e,avg}| = \frac{\mu_o wdN^2 I_{rms}^2}{3x^2} = F_{spring} \implies I_{rms} = \sqrt{\frac{3x^2 F_{spring}}{\mu_o wdN^2}} \approx 0.451 \text{ A}$$
Problem 4 (20 Points)

A doubly-excited electromechanical system with one translational variable of motion is characterized by the following flux-current relationships:

\[
\begin{align*}
\lambda_1 &= x^2i_1 + xi_2 + i_1 \\
\lambda_2 &= x^2i_2 + xi_1 + i_2
\end{align*}
\]

(a) Is this system electrically linear?  ☐ Yes  ☑ No. (Check the correct answer.)

(b) Starting from an arbitrary deenergized state \((0,0,x_o)\), indicate below the path you would take to evaluate the magnetic coenergy \(W'_m = W'_m(i_1, i_2, x)\) at a point \((i_1, i_2, x)\):

\[
\begin{align*}
i_1' &= 0 & i_1 &= i_1 \\
i_2' &= 0 & i_2 &= i_2 \\
x' &= x_o & x' &= x
\end{align*}
\]

(c) Find the magnetic coenergy \(W'_m = W'_m(i_1, i_2, x)\).

\[
W'_m = W'_m(i_1, i_2, x) = \int_{i_1}^{i_1'} \lambda_1(i_1', 0, x) \, di_1' + \int_{i_2}^{i_2'} \lambda_2(i_1, i_2', x) \, di_2' = \int_0^{i_1} \left[ x^2i_1 + i_1' \right] \, di_1' + \int_0^{i_2} \left[ x^2i_2 + xi_1 + i_2' \right] \, di_2' = x^2i_1^3/3 + i_1^2/2 + x^2i_2^3/3 + xi_1i_2 + i_2^2/2 = x^2/3 \left( i_1^3 + i_2^3 \right) + xi_1i_2 + 1/2 \left( i_1^2 + i_2^2 \right)
\]

If coil #2 is energized before coil #1, the magnetic coenergy would be

\[
W'_m = W'_m(i_1, i_2, x) = \int_0^{i_2} \lambda_2(0, i_2', x) \, di_2' + \int_{i_1}^{i_1'} \lambda_1(i_1', i_2, x) \, di_1' = \int_0^{i_2} \left[ x^2i_2 + i_2' \right] \, di_2' + \int_0^{i_1} \left[ x^2i_1 + xi_2 + i_1' \right] \, di_1' = x^2i_2^3/3 + i_2^2/2 + x^2i_1^3/3 + xi_2i_1 + i_1^2/2 = x^2/3 \left( i_1^3 + i_2^3 \right) + xi_2i_1 + 1/2 \left( i_1^2 + i_2^2 \right)
\]

(d) Find the electromagnetic force \(F_e = F_e(i_1, i_2, x)\).

\[
F_e = F_e(i_1, i_2, x) = \frac{\partial W'_m}{\partial x} = 2x/3 \left( i_1^3 + i_2^3 \right) + i_1i_2
\]