Low-level Fall Simulation:
Child rolling sideways down stairs

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Situation

A mother reported that a foster daughter sustained fatal head injuries (subdural hematomas) from tumbling down a flight of 6 carpeted stairs onto a laminate wood landing. No fractures or external bruises were reported. However, 6 bruises around the head on the inside of the scalp were revealed at autopsy. The girl was 31-months old, 37-inches tall, and weighed 33 pounds.

At issue is whether such injuries could have resulted from a chaotic tumble down the stairs. The following depiction of possible sideways rolling illustrates the situation.

Interactive Physics™ Simulation

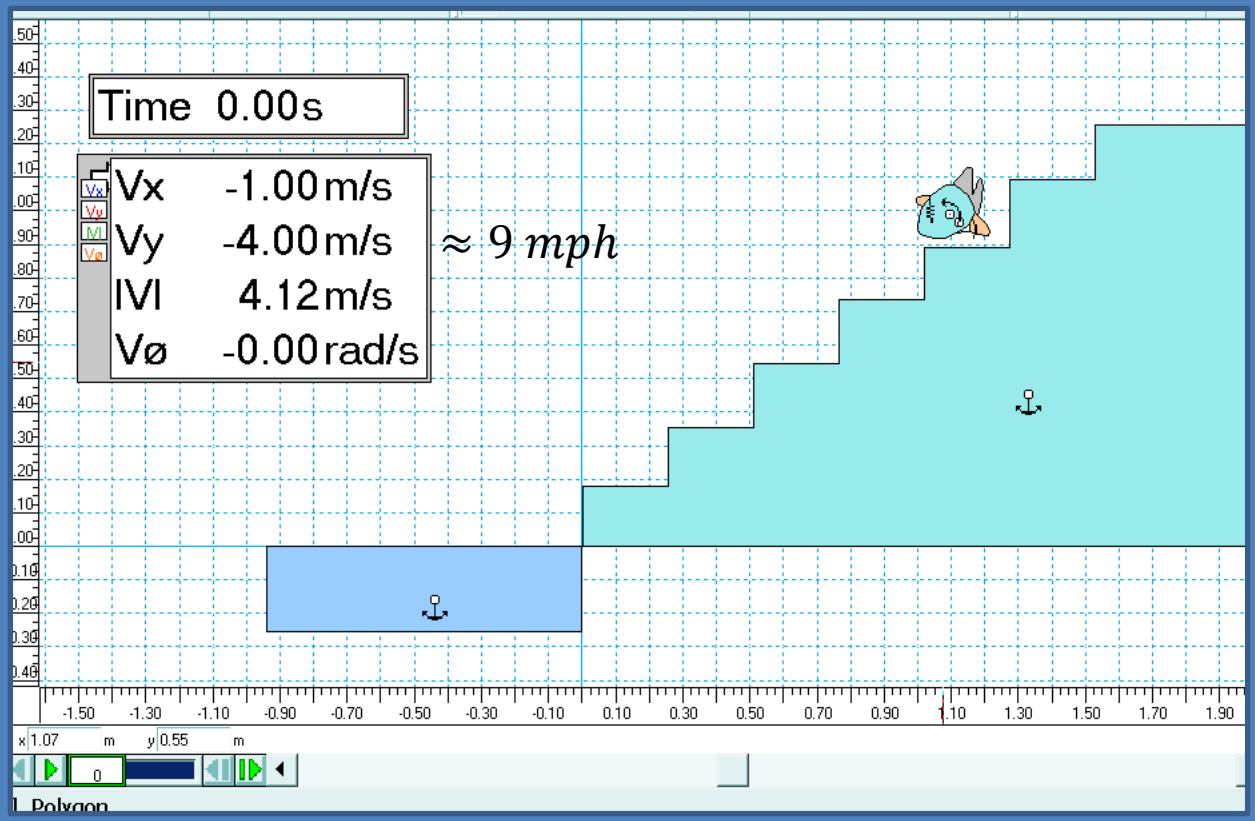
The following simulation presents an end-view of a 3-part body (head, shoulders with upper torso, and lower torso with legs). Rotational joints connect the parts to allow flexibility and energy loss. The initial conditions are chosen as reasonable approximations in the aftermath of a stumble from the top stair. The resulting fall sequence shows only one of countless possible scenarios. It is intended as a demonstration aid that is more realistic than an artist's conception.

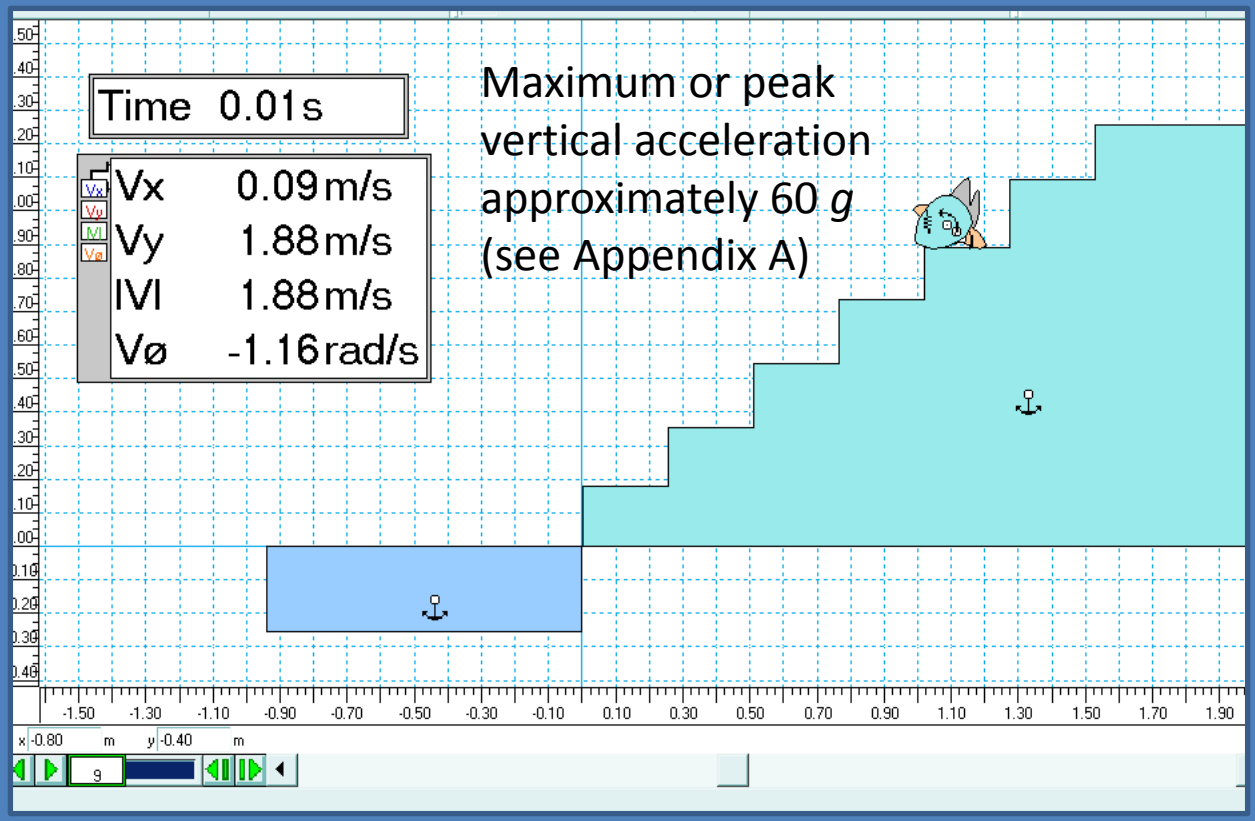
- The simulation is based on average dimensions for a 36-month old child crash-test dummy.[1] It has a stature of 0.945 m (37.2 inches) and mass of 16.2 kg (35.65 pound), including a 2.7 kg (6.0 pound) head. More details are given in Appendix B.

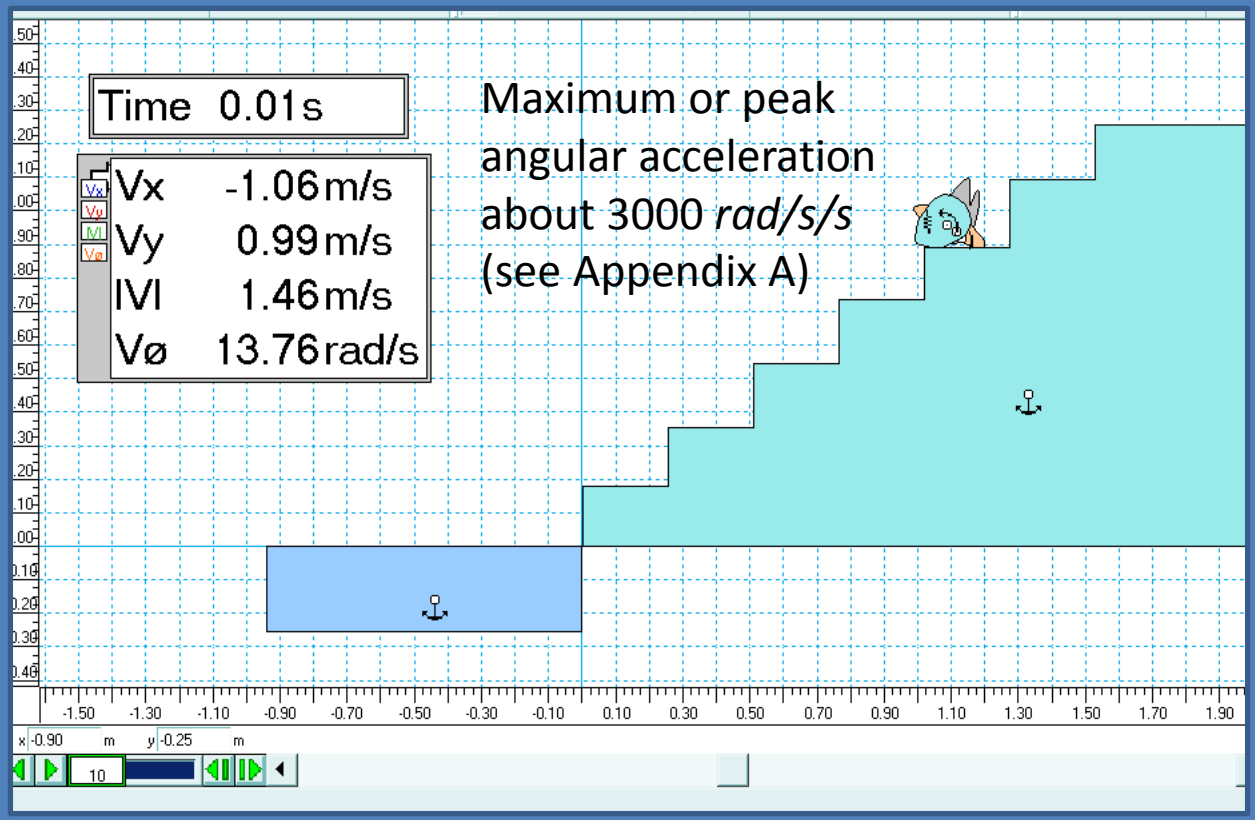
- Scale dimensions are in meters. Velocities are displayed for the head center-of-mass. Multiply values in m/s by 2.24 to convert into miles/hour.

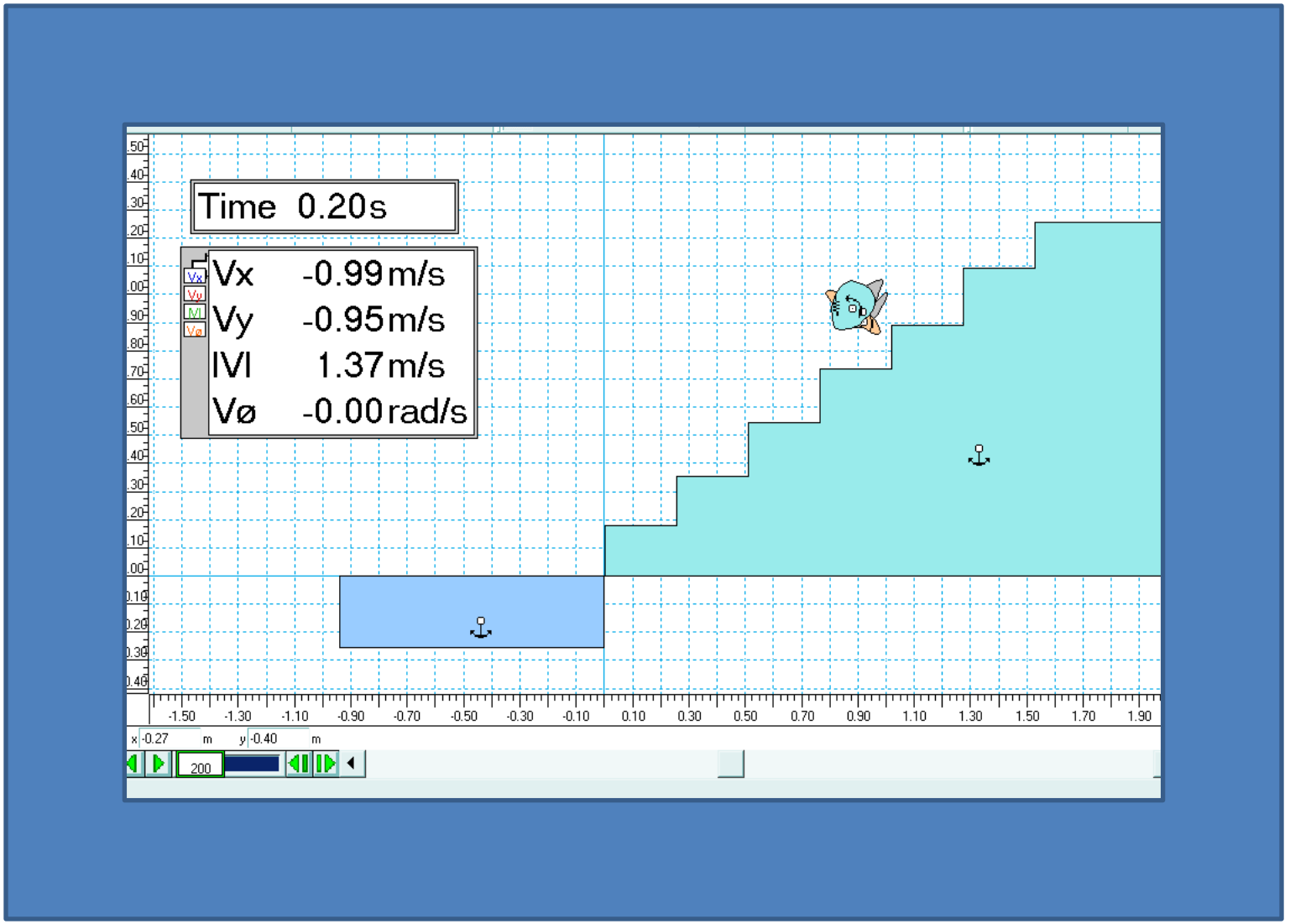
- The integrity of the software was checked by generating accurate predictions for cases with known analytical solutions. These were free-fall for a ball and toppling a long thin vertical rod about an ideal axis fixed at the bottom.

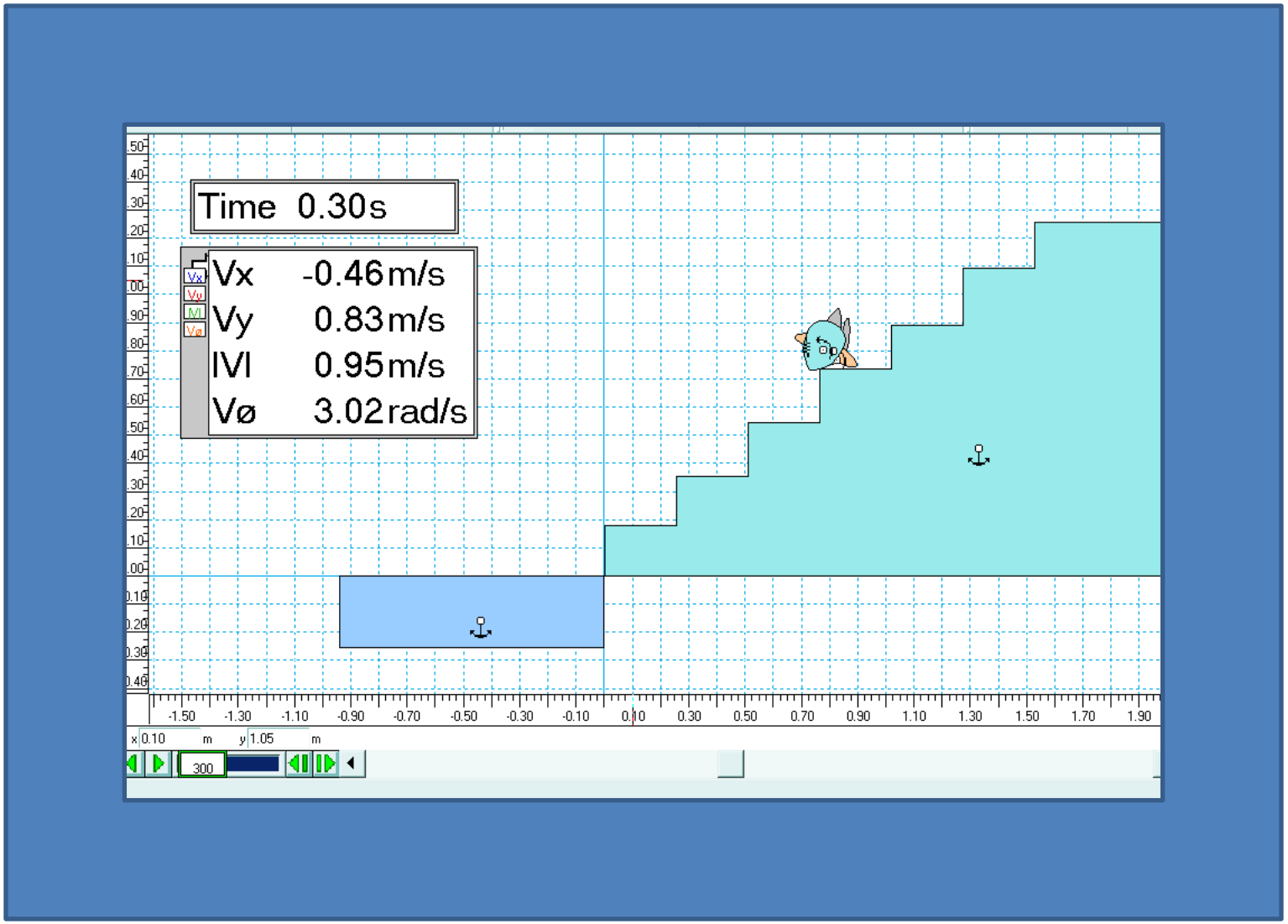
- Other computer simulations, physical models and video analysis are invited for comparison to help evaluate this simulation.

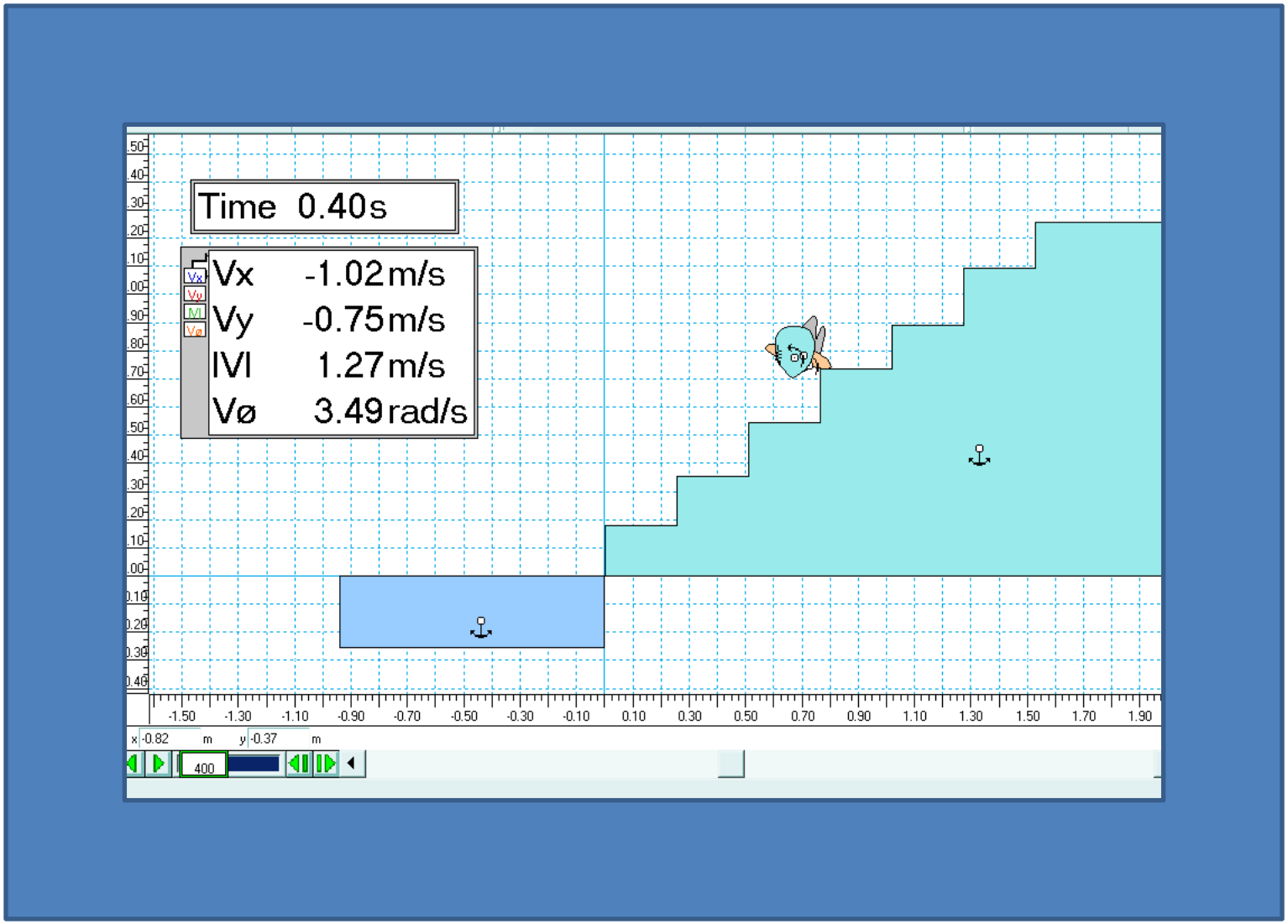

Time 0.00s
Vx -1.00 m/s
Vy -4.00 m/s
|V| 4.12 m/s
Vø -0.00 rad/s
≈ 9 mph
x 1.07 m
y 0.55 m
Polygon

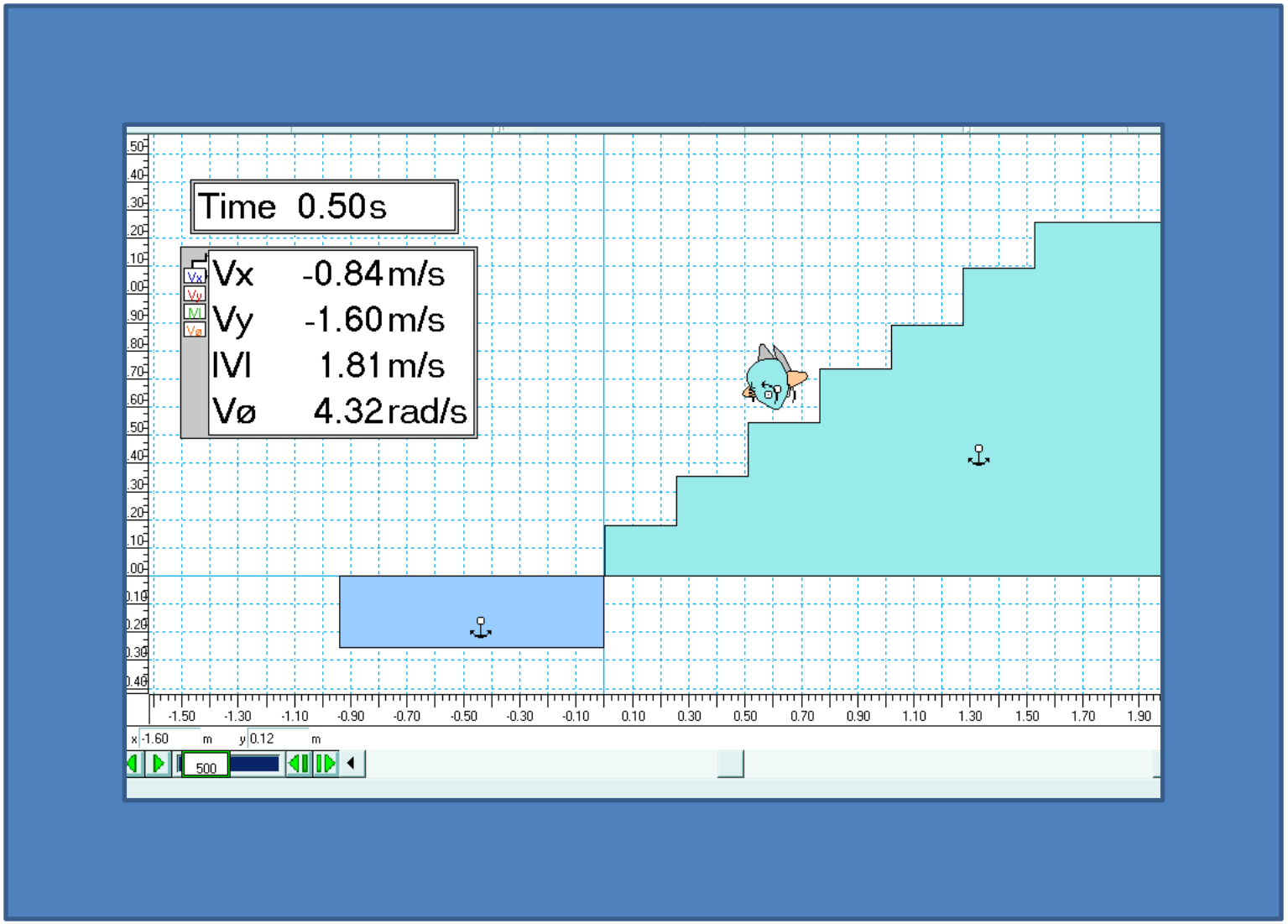

Time 0.01s
Vx 0.09 m/s
Vy 1.88 m/s
|V| 1.88 m/s
Vø -1.16 rad/s
Maximum or peak vertical acceleration approximately 60 *g* (see Appendix A)

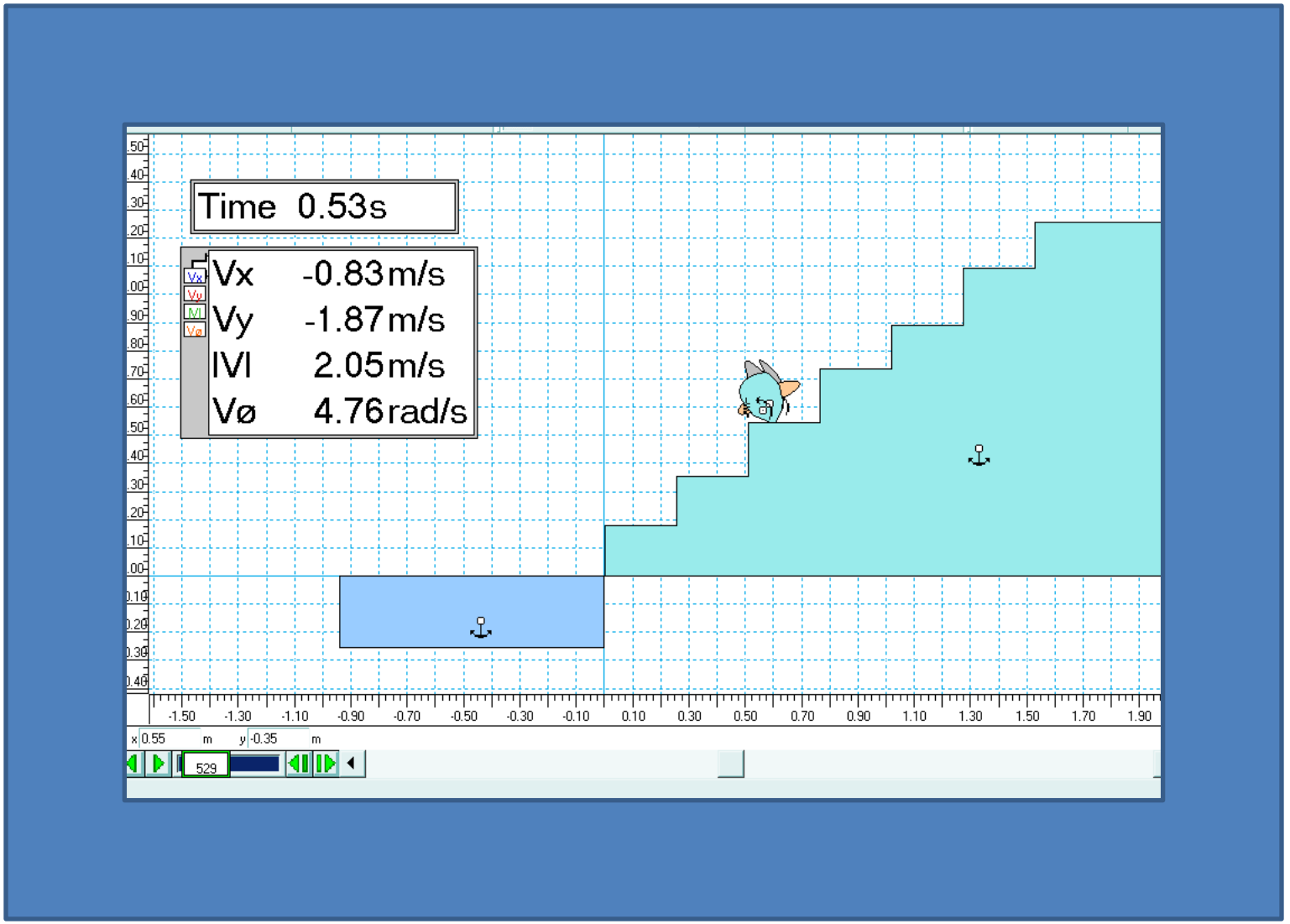

Time 0.01s
Vx -1.06 m/s
Vy 0.99 m/s
|V| 1.46 m/s
Vø 13.76 rad/s
Maximum or peak
angular acceleration
about 3000 *rad/s/s*
(see Appendix A)

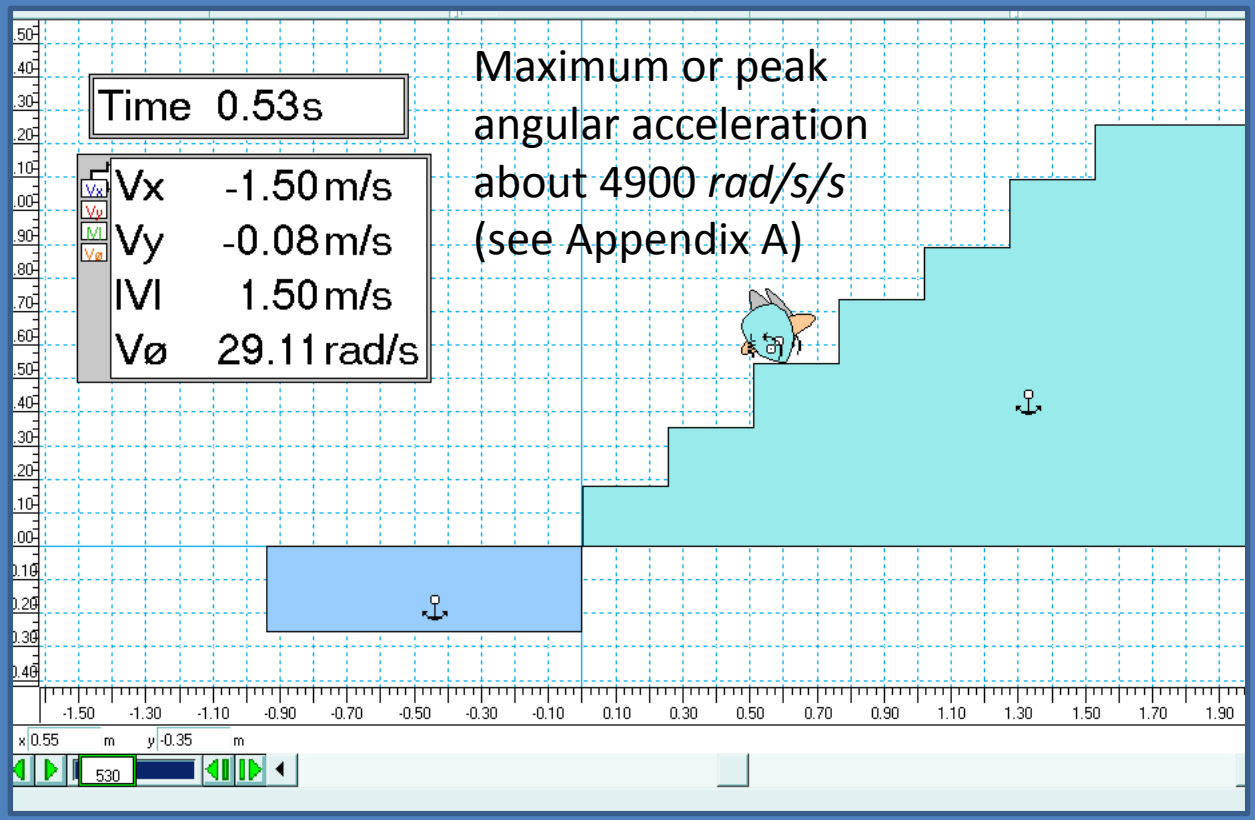

Time 0.20s
Vx -0.99 m/s
Vy -0.95 m/s
|V| 1.37 m/s
Vø -0.00 rad/s
x -0.27 m
y -0.40 m
200

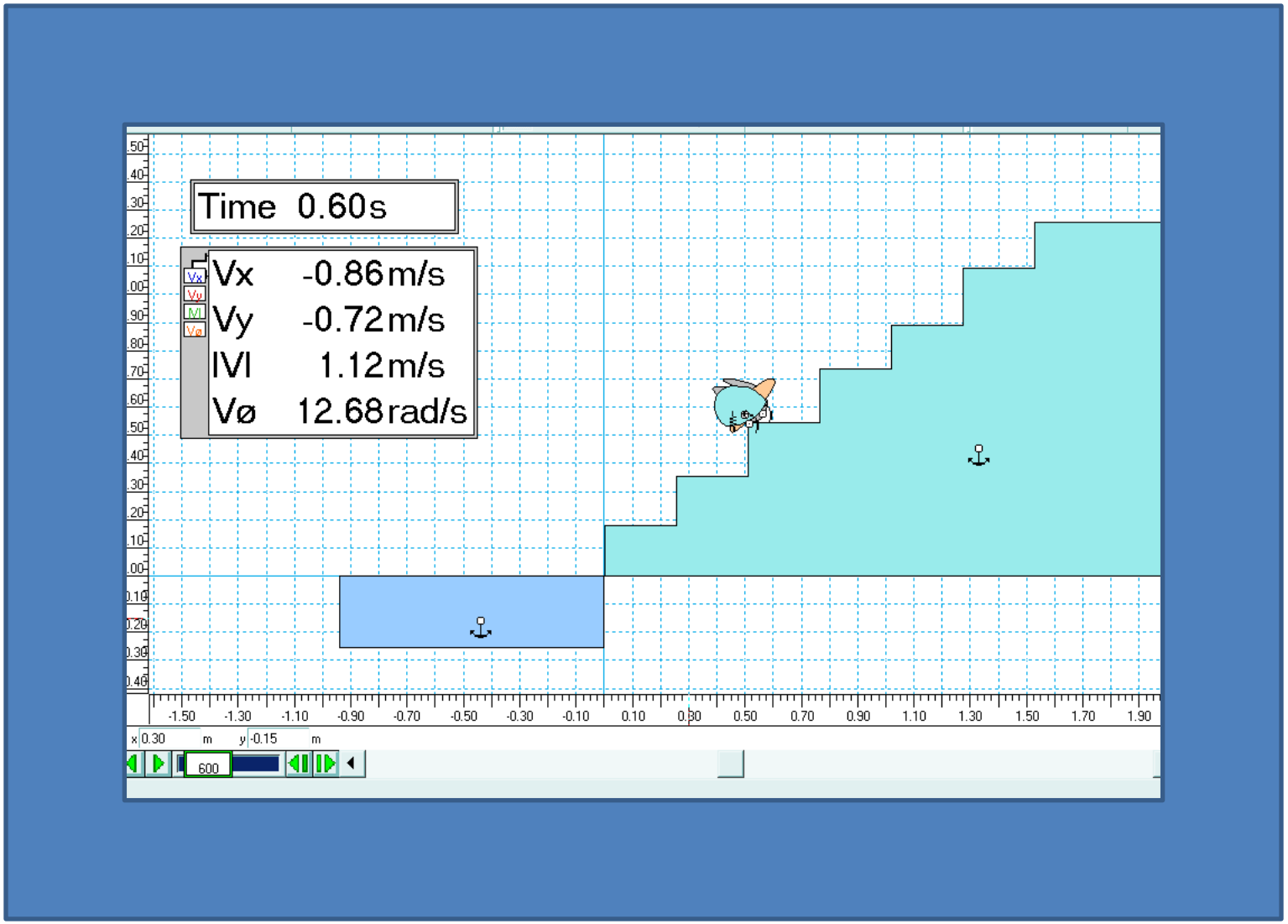

Time 0.30s
Vx -0.46 m/s
Vy 0.83 m/s
|V| 0.95 m/s
Vø 3.02 rad/s
x 0.10 m
y 1.05 m
300

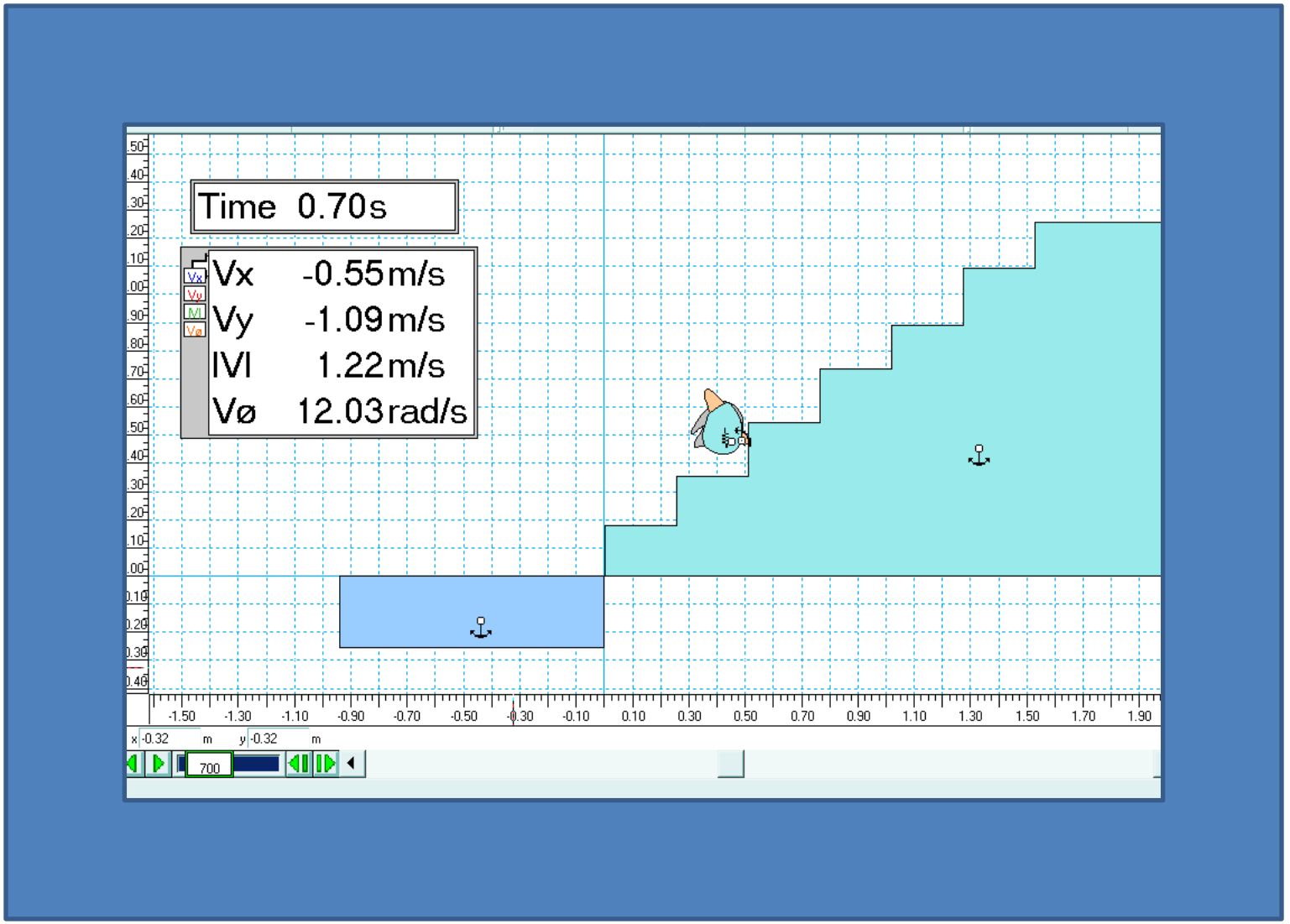

Time 0.40s
Vx -1.02 m/s
Vy -0.75 m/s
|V| 1.27 m/s
Vø 3.49 rad/s
x -0.82 m
y -0.37 m
400

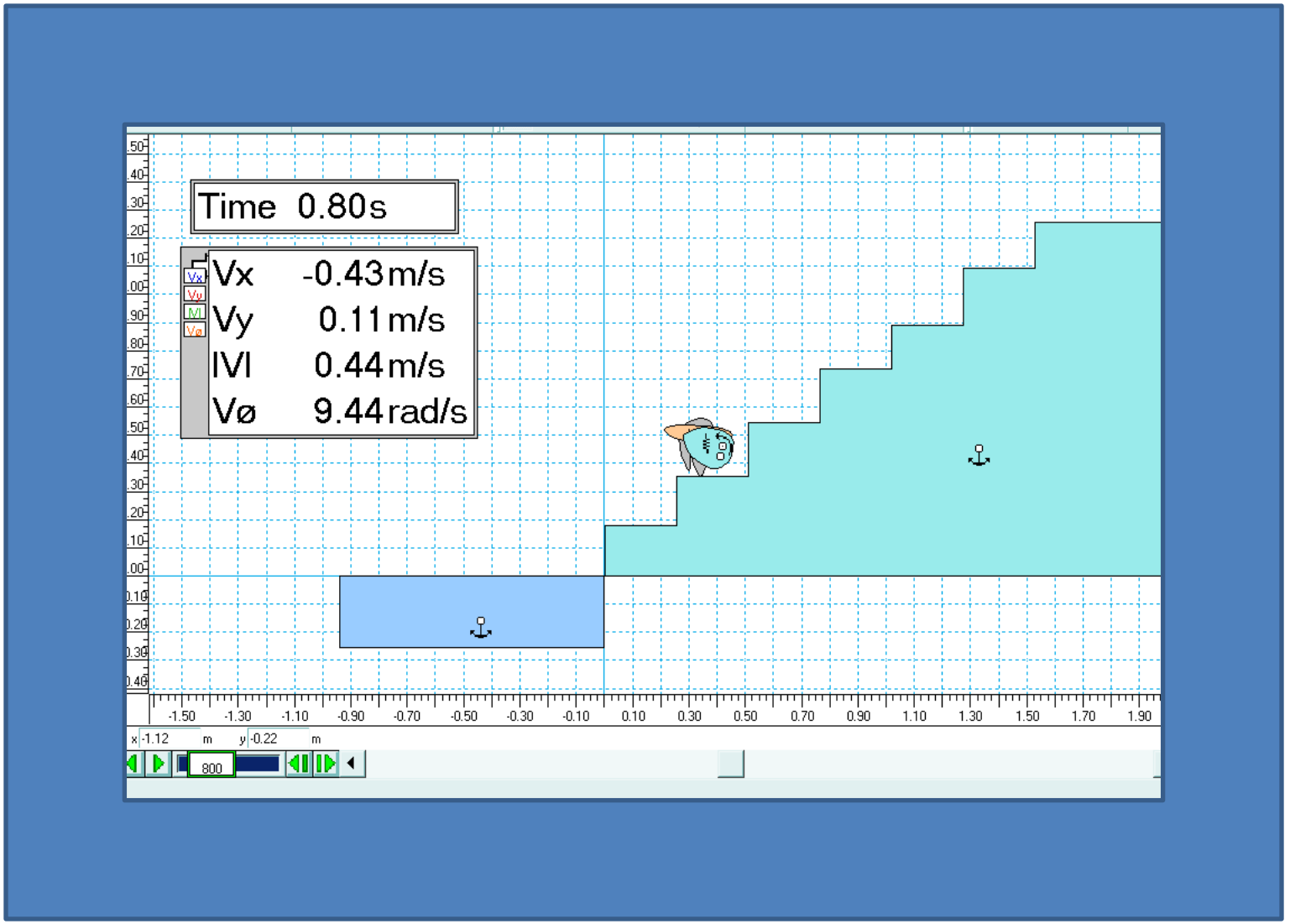

Time 0.50s
Vx -0.84 m/s
Vy -1.60 m/s
|V| 1.81 m/s
Vø 4.32 rad/s
x -1.60 m
y 0.12 m
500

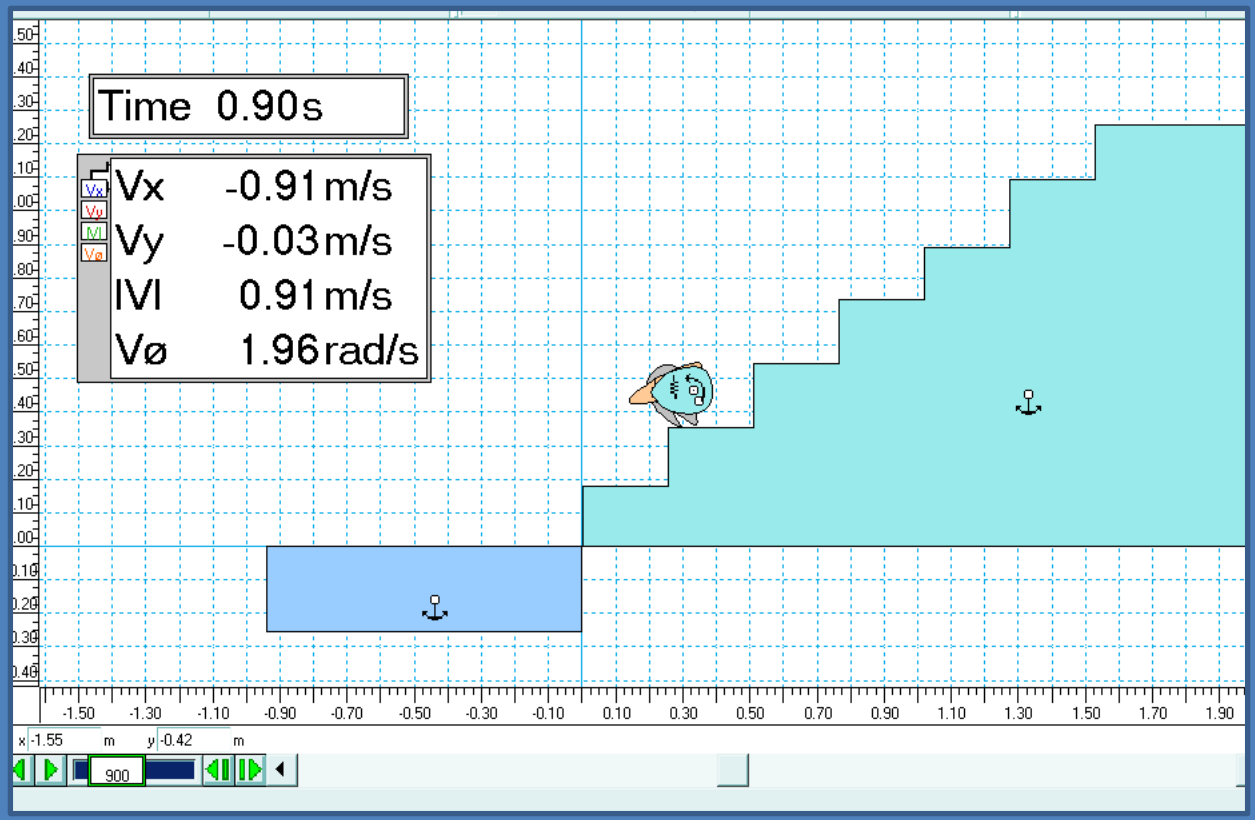

Time 0.53s
Vx -0.83 m/s
Vy -1.87 m/s
|V| 2.05 m/s
Vø 4.76 rad/s
-1.50 -1.30 -1.10 -0.90 -0.70 -0.50 -0.30 -0.10 0.10 0.30 0.50 0.70 0.90 1.10 1.30 1.50 1.70 1.90
x 0.55 m
y -0.35 m
529

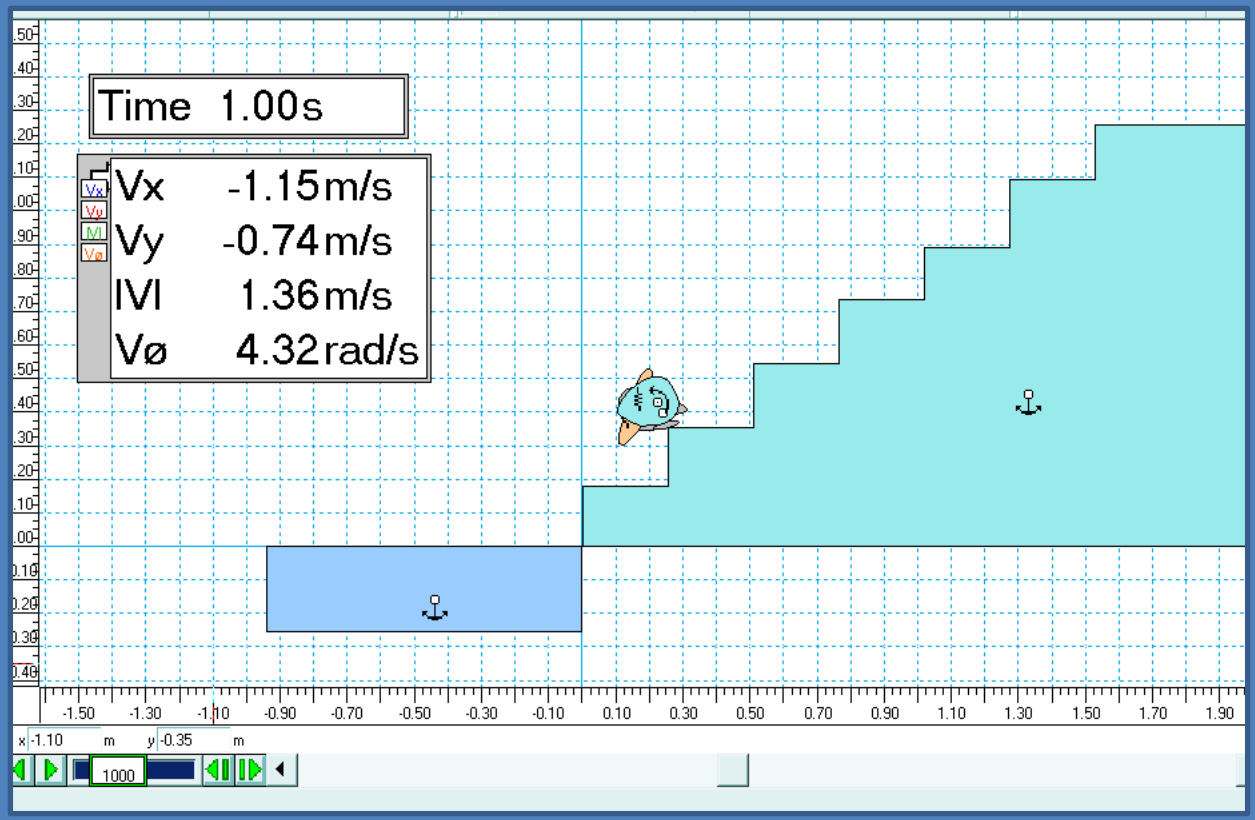

Time 0.53s
Vx -1.50 m/s
Vy -0.08 m/s
|V| 1.50 m/s
Vø 29.11 rad/s
Maximum or peak
angular acceleration
about 4900 *rad/s/s*
(see Appendix A)

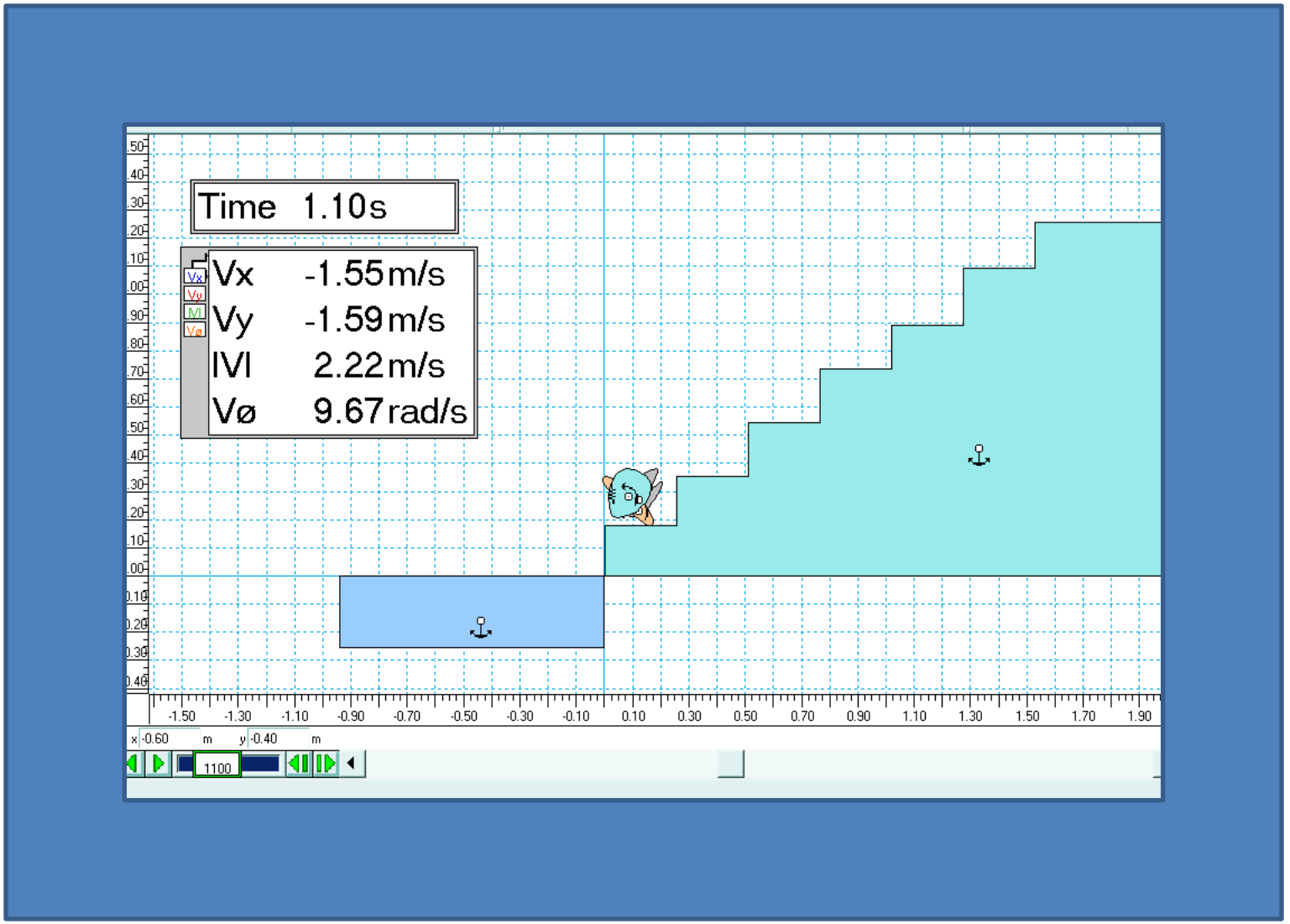

Time 0.60s
Vx -0.86 m/s
Vy -0.72 m/s
|V| 1.12 m/s
Vø 12.68 rad/s
-1.50 -1.30 -1.10 -0.90 -0.70 -0.50 -0.30 -0.10 0.10 0.30 0.50 0.70 0.90 1.10 1.30 1.50 1.70 1.90
x 0.30 m y -0.15 m
600

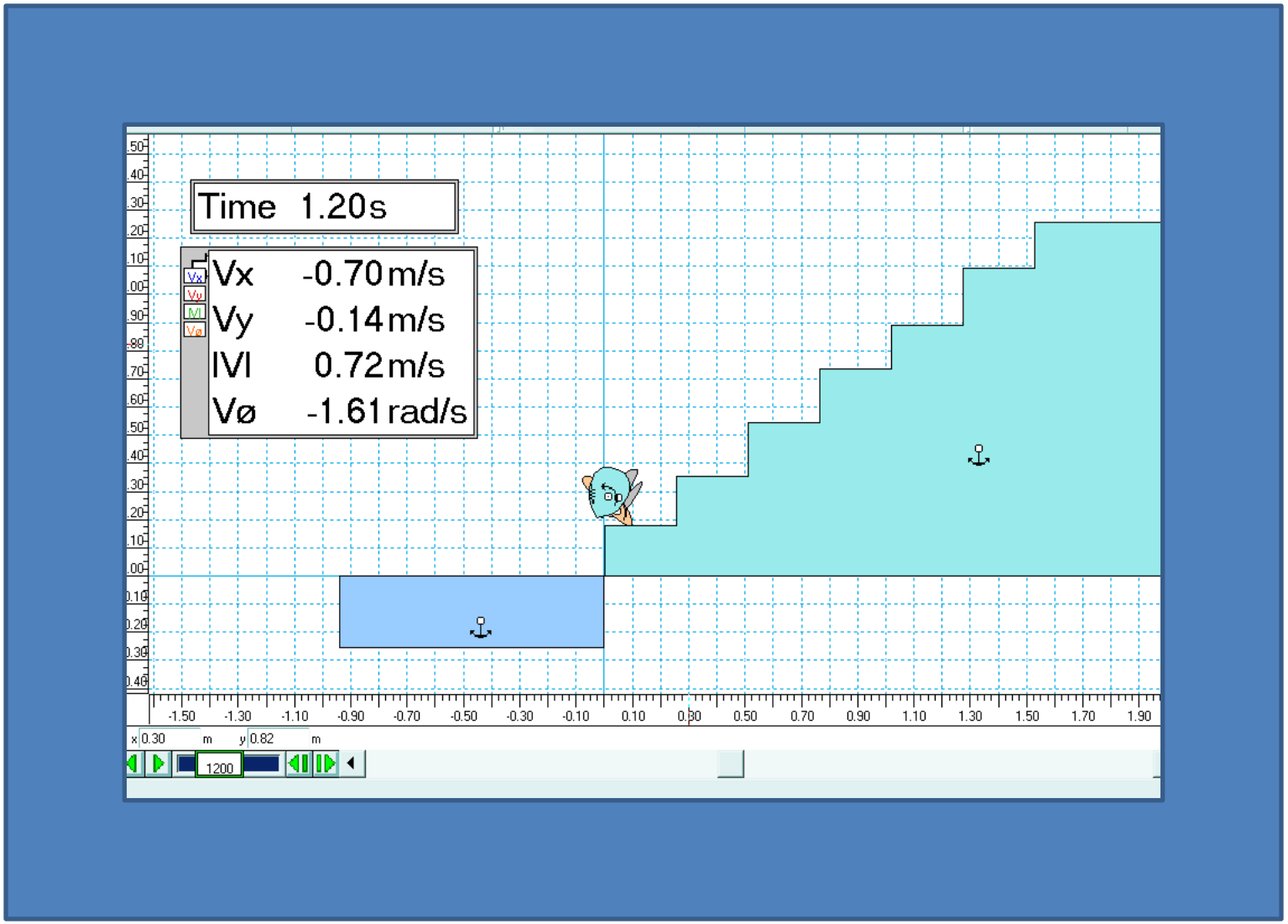

Time 0.70s
Vx -0.55m/s
Vy -1.09m/s
|V| 1.22m/s
Vø 12.03rad/s
-1.50 -1.30 -1.10 -0.90 -0.70 -0.50 -0.30 -0.10 0.10 0.30 0.50 0.70 0.90 1.10 1.30 1.50 1.70 1.90
x -0.32 m y -0.32 m
700

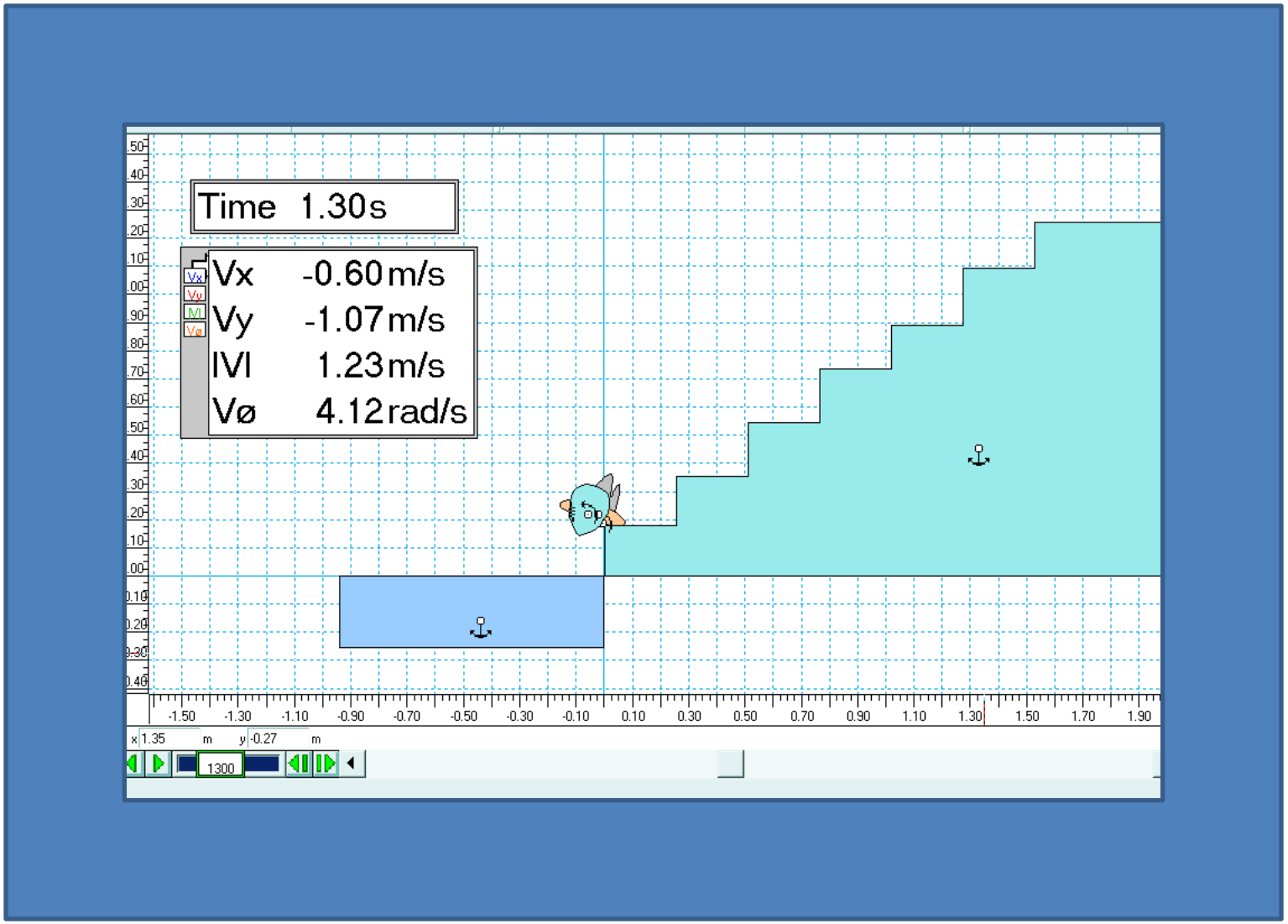

Time 0.80s
Vx -0.43 m/s
Vy 0.11 m/s
|V| 0.44 m/s
Vø 9.44 rad/s
x -1.12 m
y -0.22 m
800

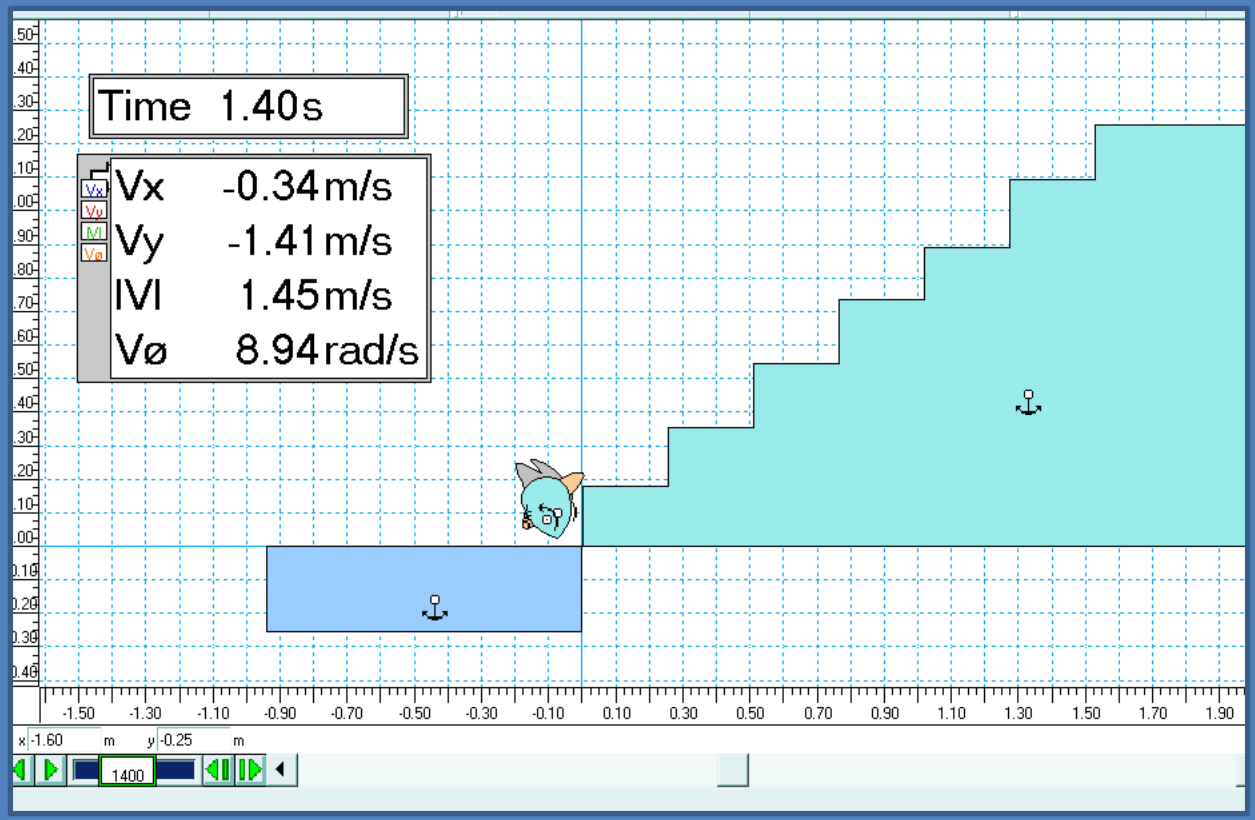

Time 0.90s
Vx -0.91 m/s
Vy -0.03 m/s
|V| 0.91 m/s
Vø 1.96 rad/s
-1.50 -1.30 -1.10 -0.90 -0.70 -0.50 -0.30 -0.10 0.10 0.30 0.50 0.70 0.90 1.10 1.30 1.50 1.70 1.90
x -1.55 m y -0.42 m
900

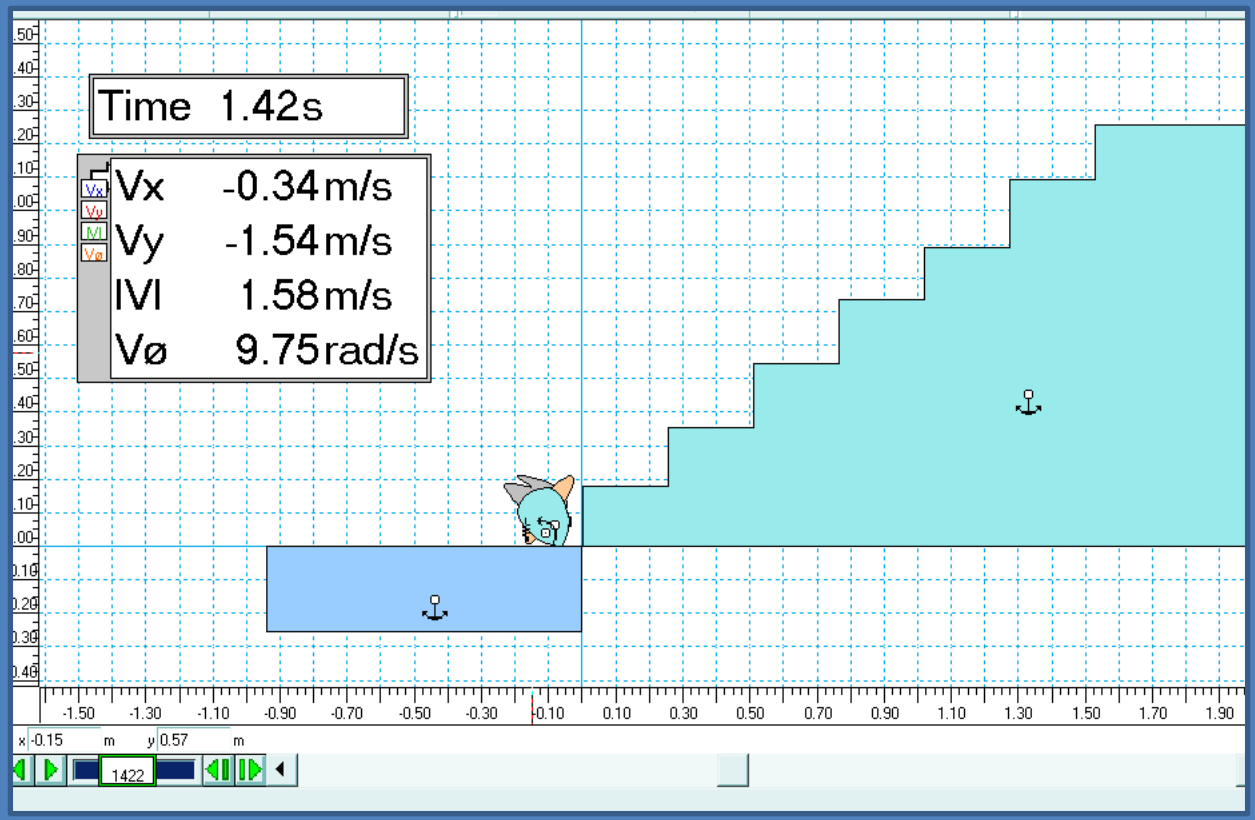

Time 1.00s
Vx -1.15m/s
Vy -0.74m/s
|V| 1.36m/s
Vø 4.32rad/s
x -1.10 m
y -0.35 m
1000

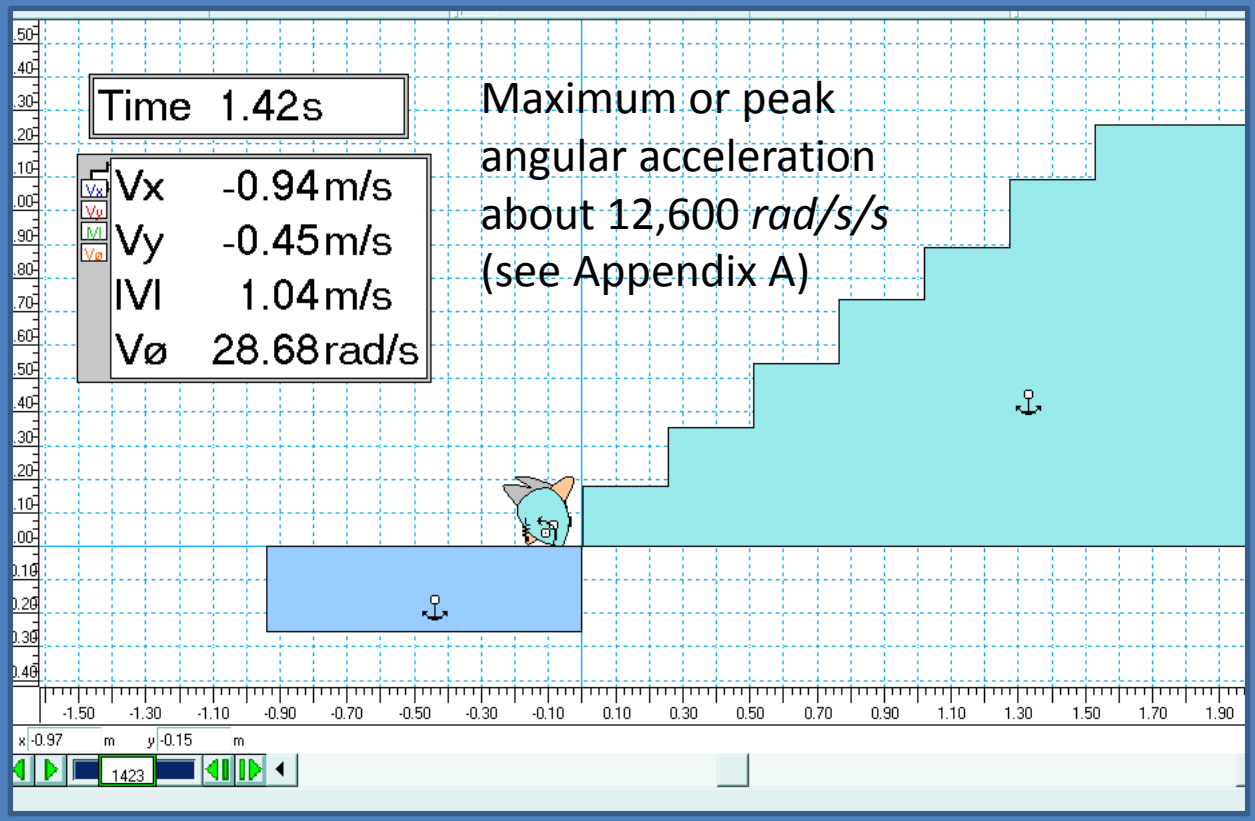

Time 1.10s
Vx -1.55 m/s
Vy -1.59 m/s
|V| 2.22 m/s
Vø 9.67 rad/s
x -0.60 m
y -0.40 m
1100

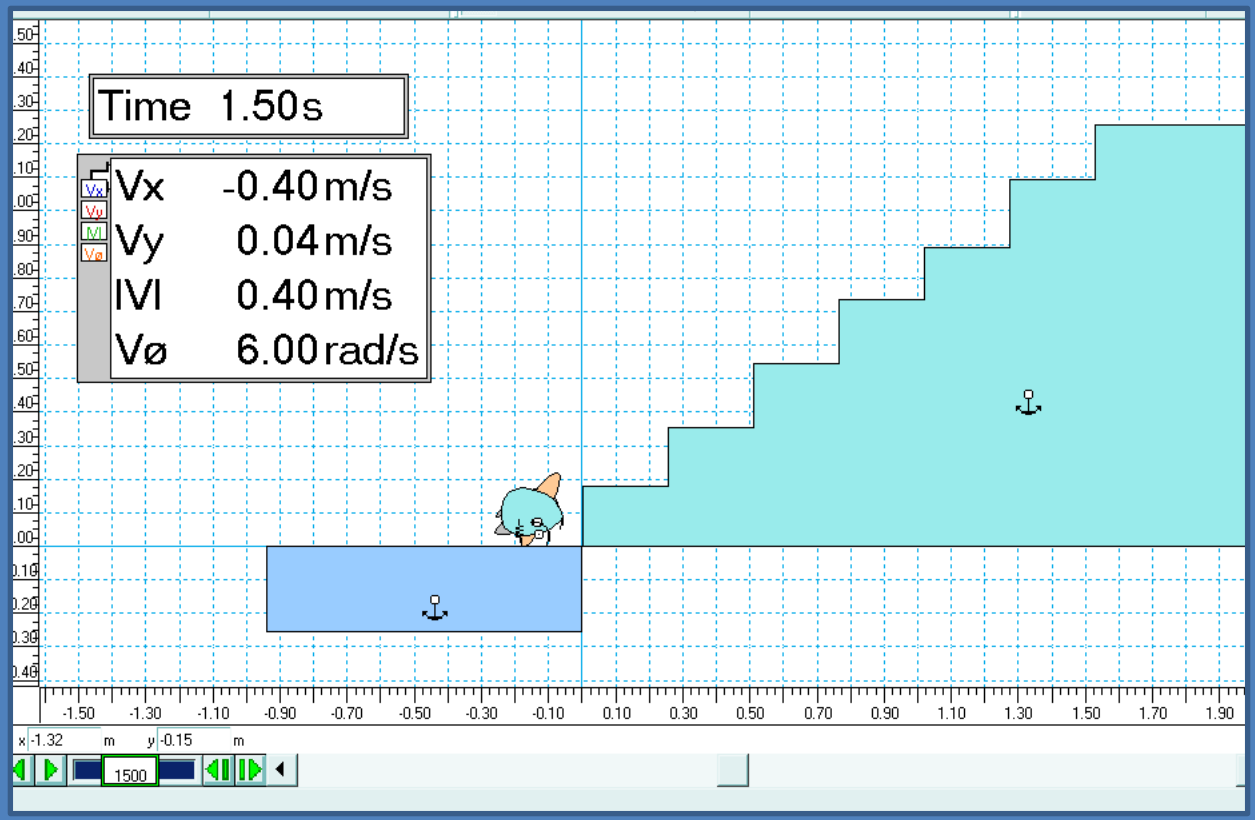

Time 1.20s
Vx -0.70 m/s
Vy -0.14 m/s
|V| 0.72 m/s
Vø -1.61 rad/s
x 0.30 m
y 0.82 m
1200

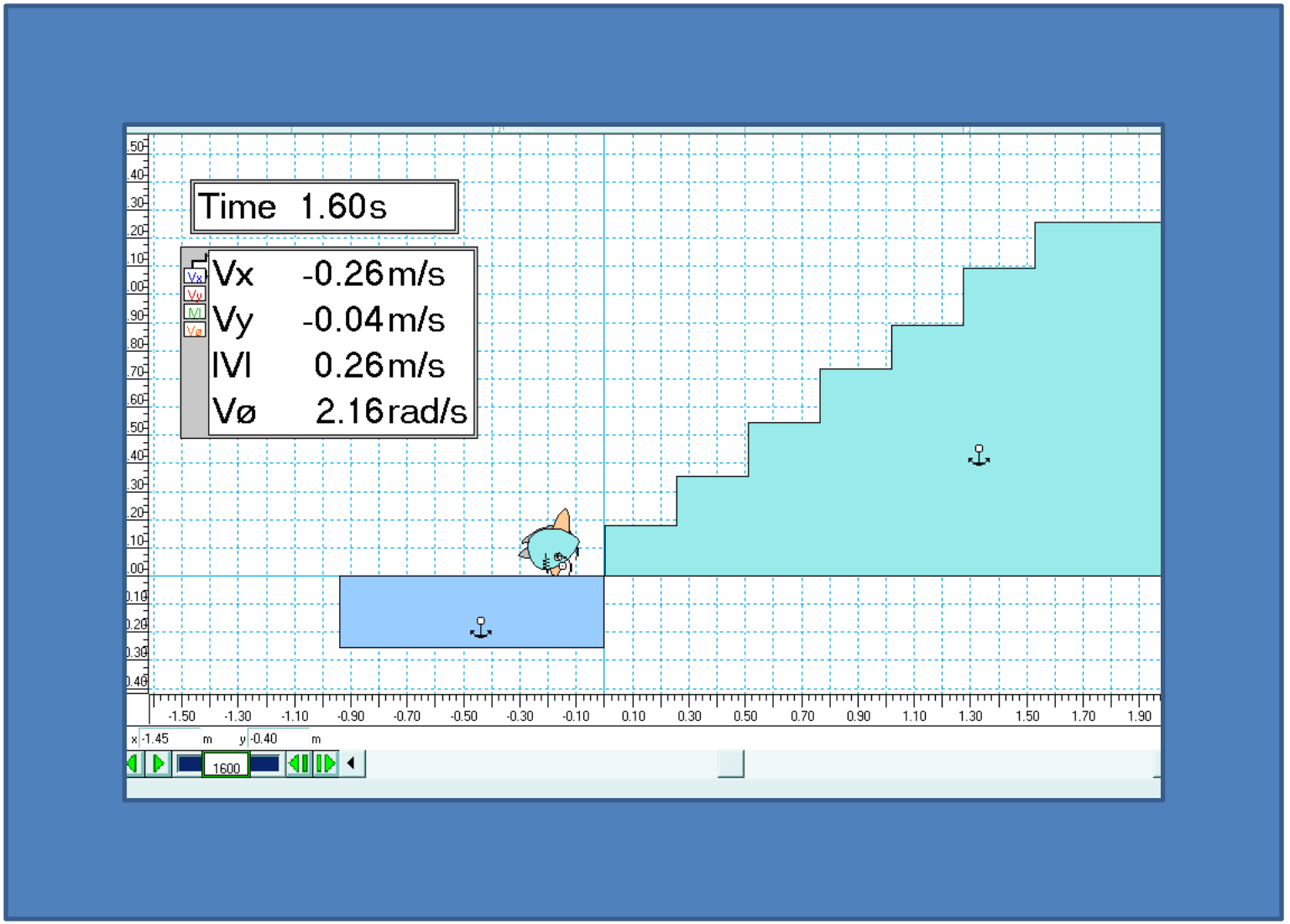

Time 1.30s
Vx -0.60 m/s
Vy -1.07 m/s
|V| 1.23 m/s
Vø 4.12 rad/s
x 1.35 m
y -0.27 m
1300

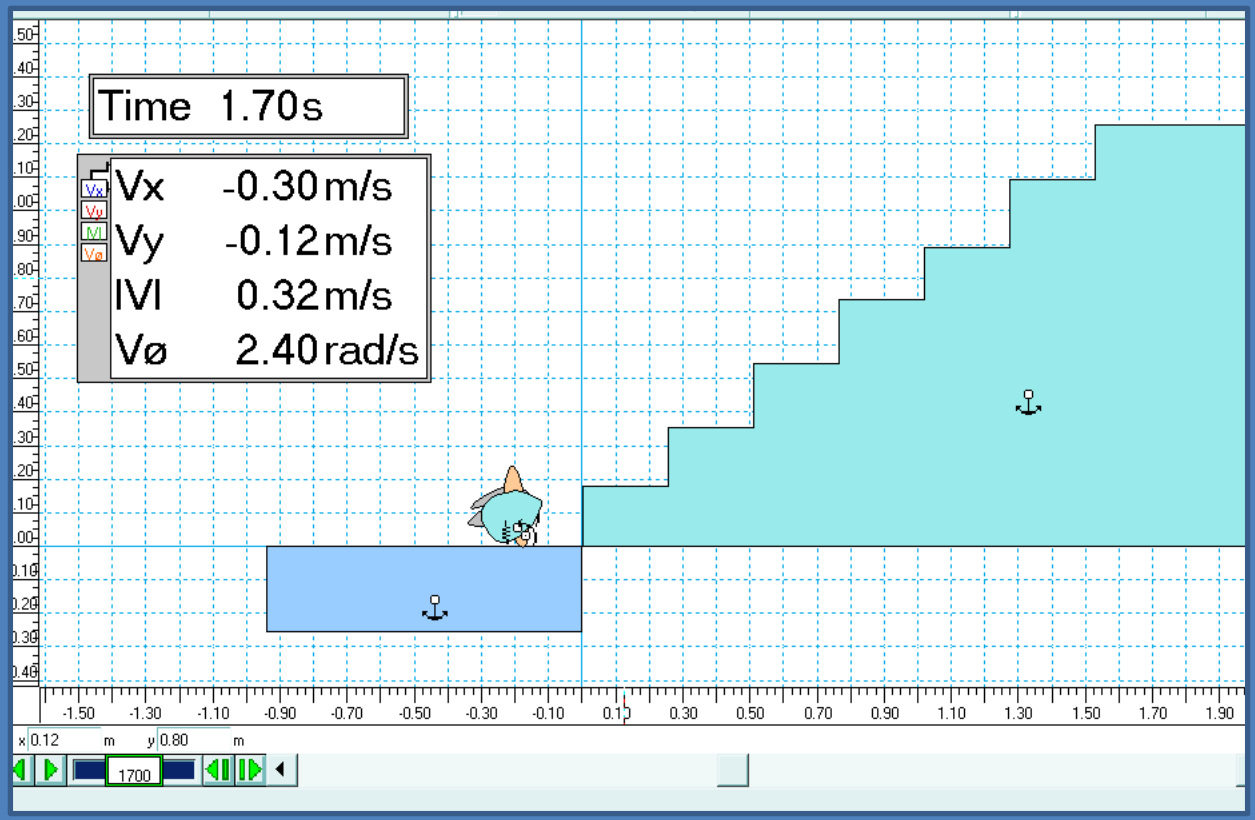

Time 1.40s
Vx -0.34 m/s
Vy -1.41 m/s
|V| 1.45 m/s
Vø 8.94 rad/s
x -1.60 m
y -0.25 m
1400

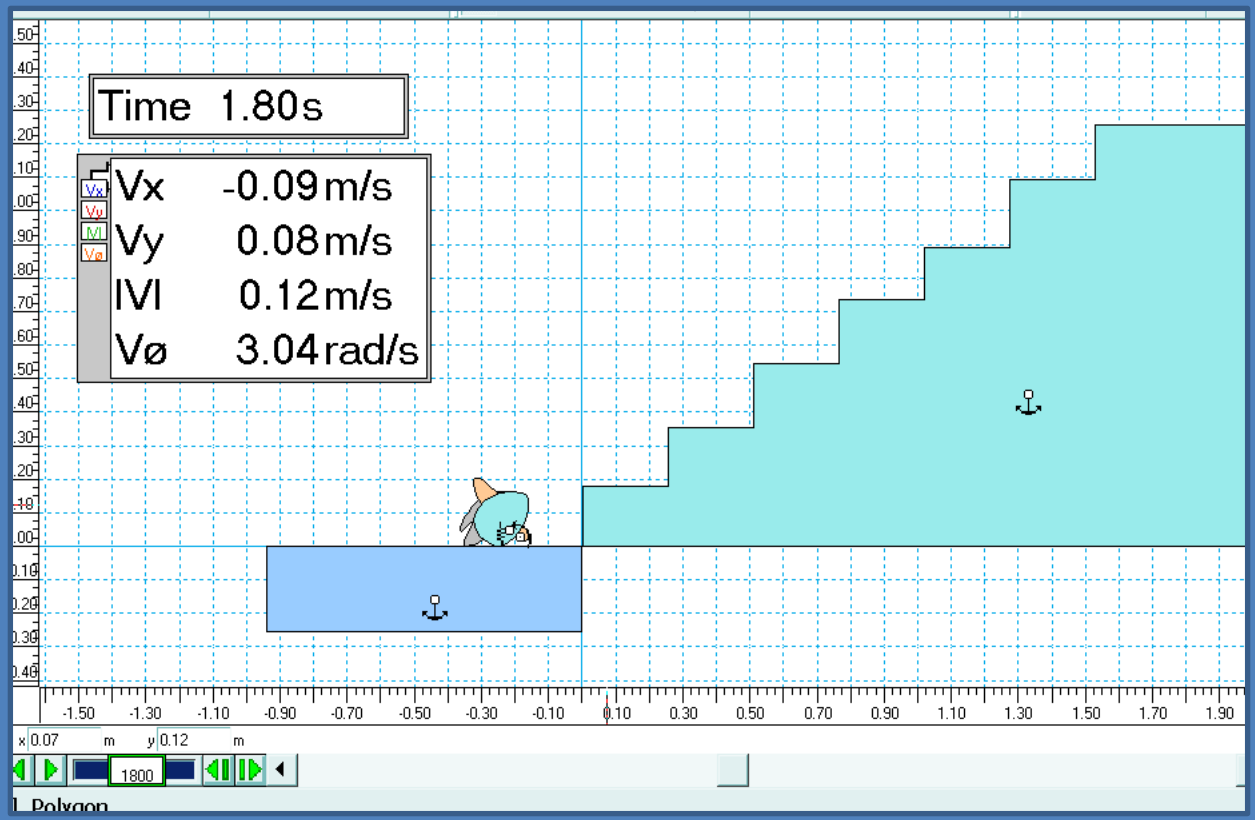

Time 1.42s
Vx -0.34 m/s
Vy -1.54 m/s
|V| 1.58 m/s
Vø 9.75 rad/s
-1.50 -1.30 -1.10 -0.90 -0.70 -0.50 -0.30 -0.10 0.10 0.30 0.50 0.70 0.90 1.10 1.30 1.50 1.70 1.90
x -0.15 m y 0.57 m
1422

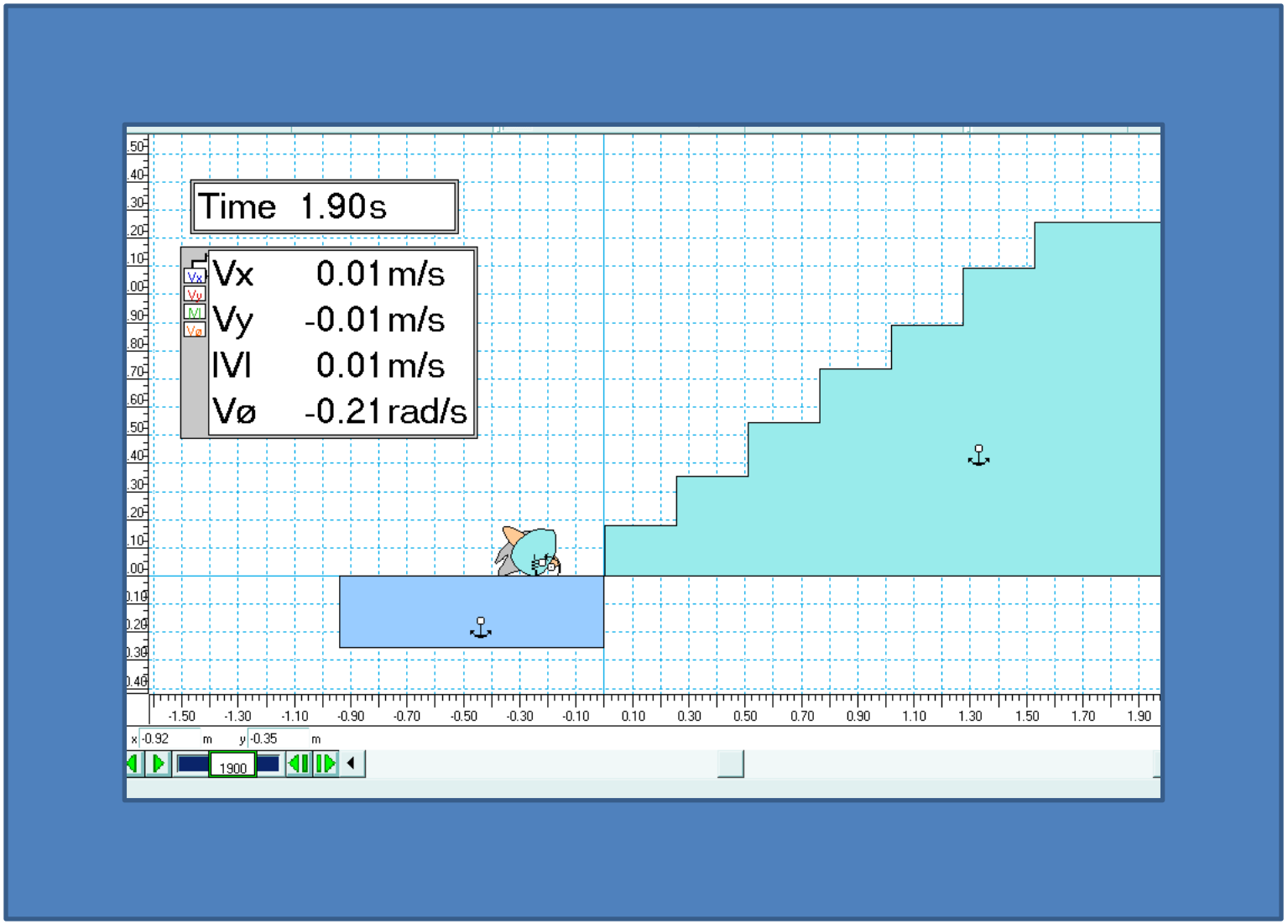

Time 1.42s
Vx -0.94 m/s
Vy -0.45 m/s
|V| 1.04 m/s
Vø 28.68 rad/s
Maximum or peak angular acceleration about 12,600 *rad/s/s* (see Appendix A)
-1.50 -1.30 -1.10 -0.90 -0.70 -0.50 -0.30 -0.10 0.10 0.30 0.50 0.70 0.90 1.10 1.30 1.50 1.70 1.90
x -0.97 m y -0.15 m
1423

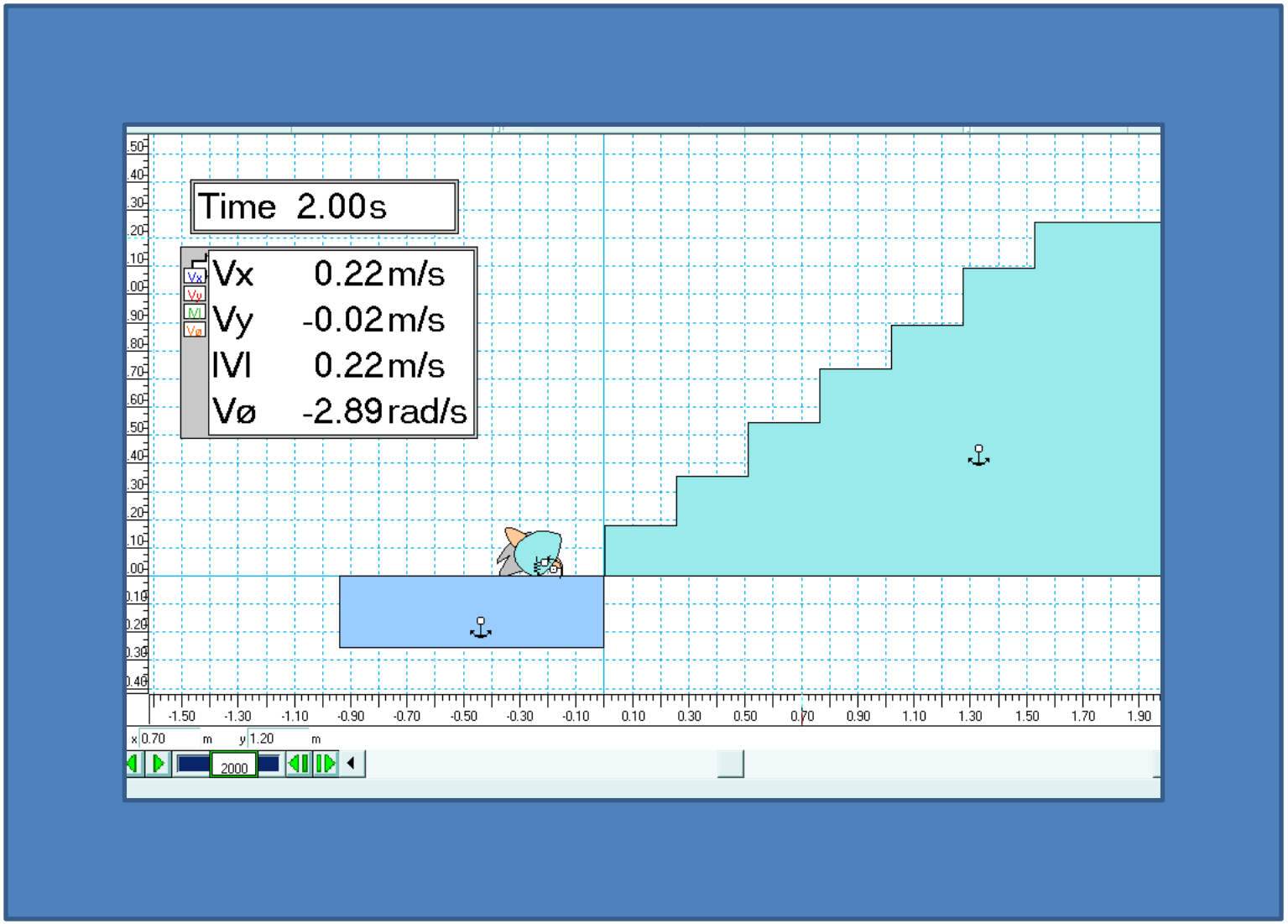

Time 1.50s
Vx -0.40 m/s
Vy 0.04 m/s
|V| 0.40 m/s
Vø 6.00 rad/s
x -1.32 m
y -0.15 m
1500

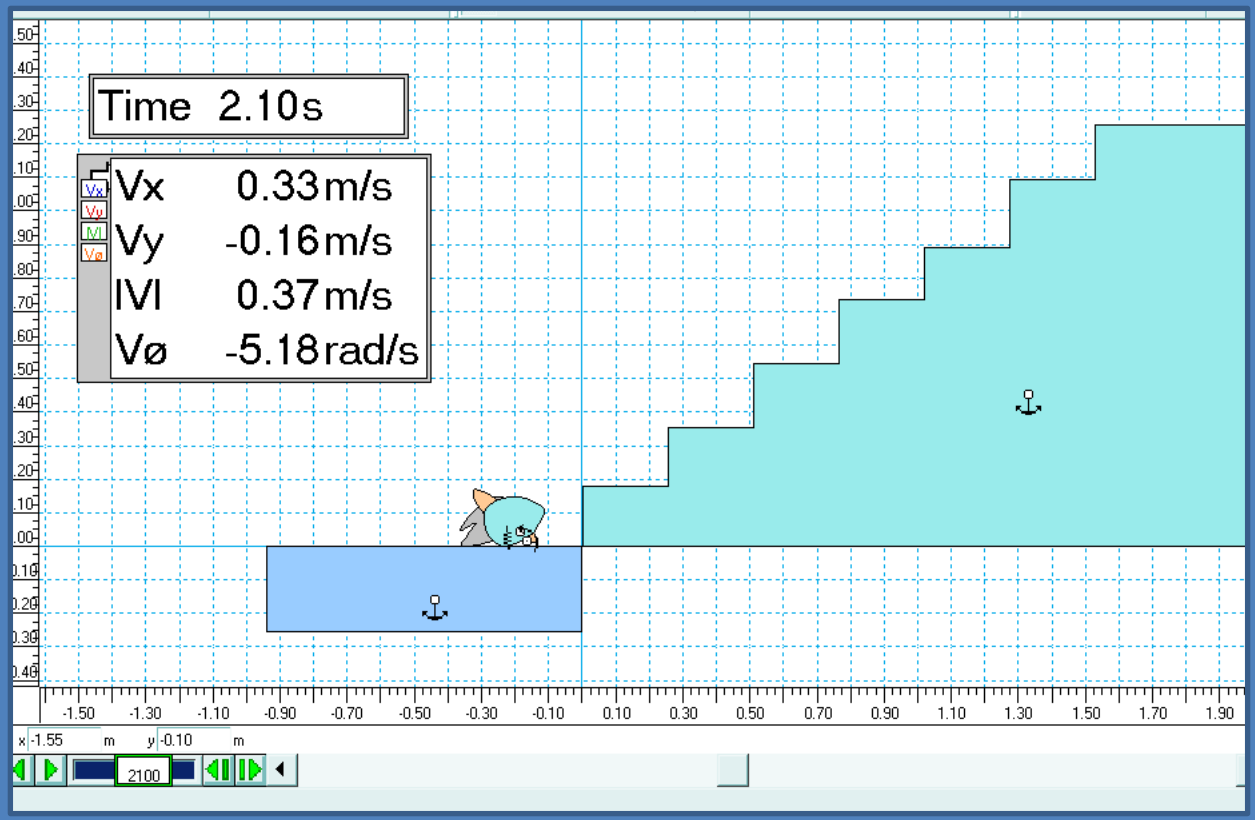

Time 1.60s
Vx -0.26 m/s
Vy -0.04 m/s
|V| 0.26 m/s
Vø 2.16 rad/s
-1.50 -1.30 -1.10 -0.90 -0.70 -0.50 -0.30 -0.10 0.10 0.30 0.50 0.70 0.90 1.10 1.30 1.50 1.70 1.90
x -1.45 m y -0.40 m
1600

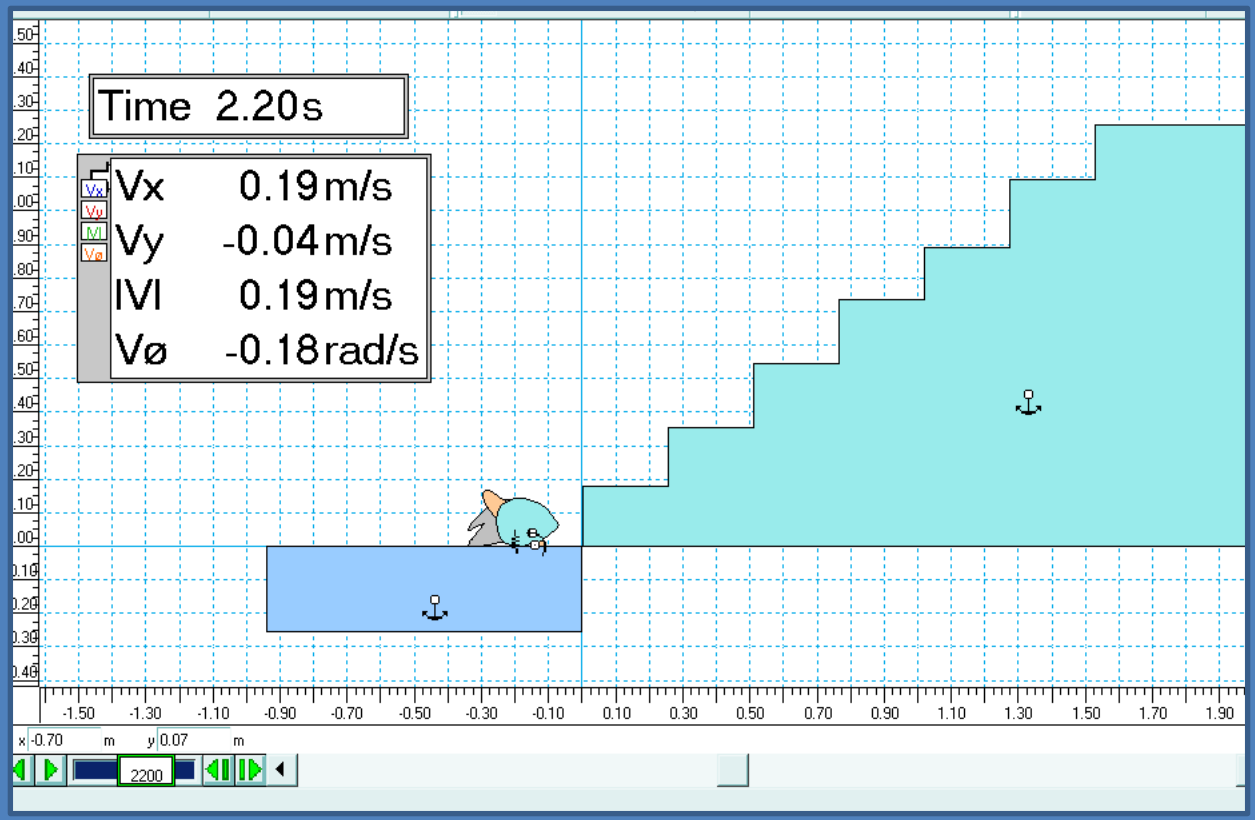

Time 1.70s
Vx -0.30 m/s
Vy -0.12 m/s
|V| 0.32 m/s
Vø 2.40 rad/s
x 0.12 m
y 0.80 m
1700

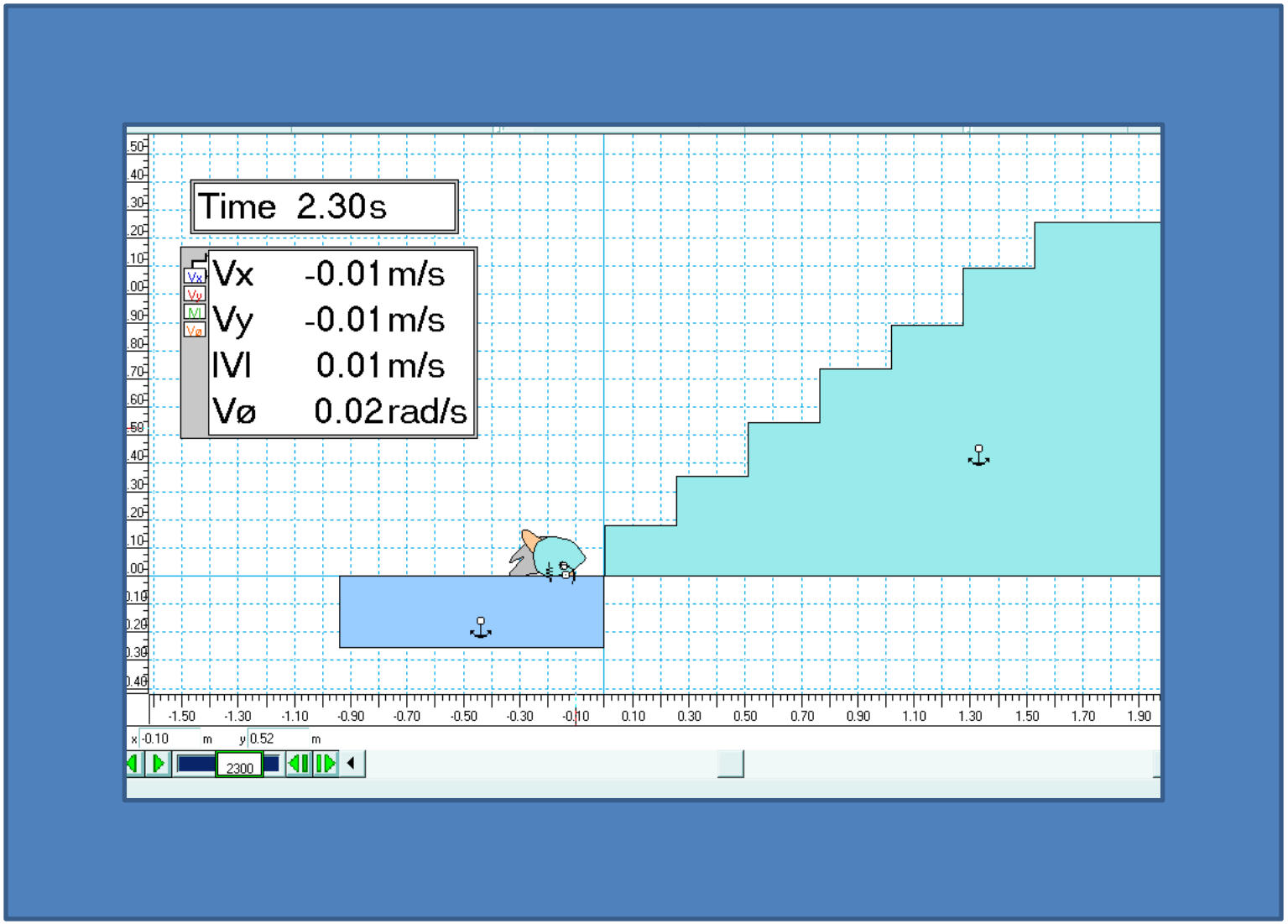

Time 1.80s
Vx -0.09 m/s
Vy 0.08 m/s
|V| 0.12 m/s
Vø 3.04 rad/s
x 0.07 m
y 0.12 m
1800
Polygon


Time 1.90s
Vx 0.01 m/s
Vy -0.01 m/s
|V| 0.01 m/s
Vø -0.21 rad/s
x -0.92 m
y -0.35 m
1900


Time 2.00s
Vx 0.22 m/s
Vy -0.02 m/s
|V| 0.22 m/s
Vø -2.89 rad/s
x 0.70 m
y 1.20 m
2000


Time 2.10s
Vx 0.33m/s
Vy -0.16m/s
|V| 0.37m/s
Vø -5.18rad/s
-1.50 -1.30 -1.10 -0.90 -0.70 -0.50 -0.30 -0.10 0.10 0.30 0.50 0.70 0.90 1.10 1.30 1.50 1.70 1.90
x -1.55 m y -0.10 m
2100


Time 2.20s
Vx 0.19 m/s
Vy -0.04 m/s
|V| 0.19 m/s
Vø -0.18 rad/s
x -0.70 m
y 0.07 m
2200


Time 2.30s
Vx -0.01 m/s
Vy -0.01 m/s
|V| 0.01 m/s
Vø 0.02 rad/s
x -0.10 m
y 0.52 m
2300

Results

This simulation clearly shows the potential for multiple bruises as the head is buffeted by contact with the stairs. The carpeting might soften the blows enough to prevent surface bruising but still mark the inside of the scalp by pressure against the hard skull. The shapes of the bruises are difficult to predict because of the skull's irregular shape and the tendency of the scalp to "pinch" when pushed sideways.

There is also potential for severe brain injuries such as subdural hematomas due to the extreme angular accelerations that are encountered.[2,3]

Conclusion

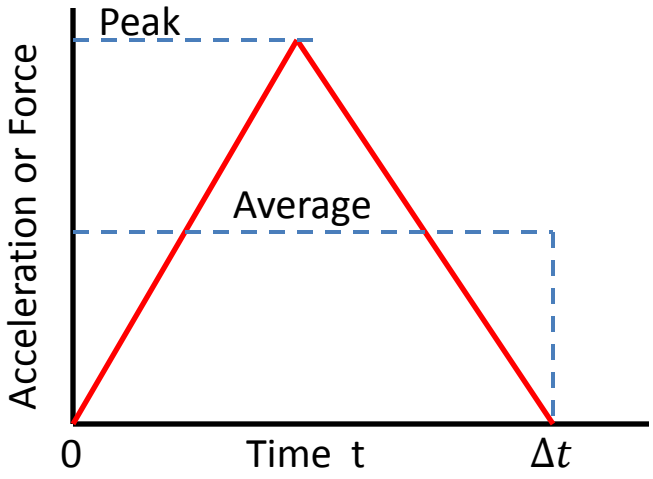
The woman's account is reasonable. The observed injuries were consistent with a chaotic sideways tumble down carpeted stairs onto a rigid landing.

Appendix A. Physics Relationships

- By definition, the average acceleration in the vertical dimension $\langle a_y \rangle \equiv \frac{\Delta v_y}{\Delta t}$, where Δv_y is the change in velocity and Δt is the interaction time with the floor.

- A toddler head colliding with a rigid surface has $\Delta t \approx 0.005\ s$.[2,3] Moderate padding extends this to $\Delta t \approx 0.020\ s$. $\Delta t = 0.006\ s$ was chosen as a reasonable value for a hardwood floor. Angular acceleration times are about half as long since rolling usually begins.

- As illustrated, a linear approximation of the acceleration or force between the floor and the head shows that the maximum or peak value is about twice the average.[4, 5]



- Divide accelerations in m/s^2 by $g = 9.81\ m/s^2$ to convert into *g's.* Multiply accelerations by mass to convert into forces.

Appendix B. Model Specifications (typical 36 month old child)[1]

Body Segment	Mass (kg)	Weight (lbs)
Head	2.72	6.0
Neck	0.79	1.74
Torso	6.99	15.4
Combined upper arms	0.88	1.94
Combined lower arms with hands	0.92	2.02
Combined upper legs	2.02	4.46
Combined lower legs	1.22	2.68
Combined feet	0.62	1.36
TOTAL	16.16 kg	35.65 lbs

Body representation used:

2.72 kg for head
7.0 kg for shoulders + upper torso
6.5 kg for lower torso + legs

Rotational spring joints used:

k = 1.0 N-m/rad, b = 0.2 N-m-s/rad for neck
k = 10 N-m/rad, b = 0.5 N-m-s/rad for waist

Elasticity or coefficient of restitution:

0.5 for the head
0.1 for other body portions.

For all frictional interactions:

static coefficient = 0.4
kinetic coefficient = 0.3

- Rotational joint values were chosen for realistic response since standards are impossible for a variable organism.
- Kutta-Merson with 0.001 second integration steps provides accuracy and model stability. Only one step out of every hundred is illustrated.

References

1. http://www.humaneticsatd.com/crash-test-dummies/children

2. Ommaya A K, Goldsmith W and Thibault L. Biomechanics and neuropathology of adult and paediatric head injury. British Journal of Neurosurgery 2002; 16(3): 220-242

3. Goldsmith W and Plunkett J. A biomechanical analysis of the causes of traumatic brain injury in infants and children. The American Journal of Forensic Medicine and Pathology June 2004; 25(2): 89-100

4. Cross, Rod. The Bounce of a Ball. American Journal of Physics March 1999; 67 (3): 222-227