PHYS 432 Extra Credit for H12 due 18 November 2013

Explain your reasoning clearly and show all work.

1. Use the Maxwell speed distribution given in Eq. (6.50) on page 244 to estimate the number of air molecules in MP-408 whose speeds are greater than $c/10$, where c is the speed of light. See Eq. (6.54) on page 245.

2. Let F be the Helmholtz free energy of Eqs. (6.57) and (6.58) on page 247, and define $\tilde{F} \equiv -kT \ln Z$, as in Eq. (6.60) on page 248.
(a) Derive Eqs. (6.59) and (6.62), showing all mathematical steps, thereby proving that F and $\tilde{F}$ both obey the same first-order differential equation with respect to T.
(b) Next, use Eq. (6.59) to show that Eq. (6.62) and the $T = 0$ boundary condition in Eq. (6.63) are solved by $\tilde{F} = F + TS_0(V, N)$, where $S_0(V, N)$ is any extensive function of the volume V and number N. Thus, the reasoning given on page 248 of the textbook does not quite prove that $\tilde{F} = F$ for all T, V, N. In order to prove that $\tilde{F} = F$ for all T, V, N, we would need to show that $S_0(V, N) = 0$ for all V and N, which you will do next in part (c) below.
(c) By taking minus the partial derivative with respect to T of both sides of the equation $\tilde{F} = F + TS_0(V, N)$ that you proved in part (b) above, with V, N held fixed, and then taking the zero-temperature limit $T \to 0$, use Eq. (6.58) on page 247 to show that the extensive function $S_0(V, N)$ from part (b) above is equal to zero if the zero-temperature limit of $S(T, V, N)$ is equal to the residual entropy described on the bottom half of page 94: i.e., if

$$\lim_{T \to 0} S(T, V, N) = k \ln \Omega_0(V, N),$$

where $\Omega_0(V, N)$ is the ground-state (or zero-temperature) degeneracy (or multiplicity) of the system. *Hint*: consider the zero-temperature limit of Eq. (6.12).
Thus parts (a), (b), and (c) taken together prove Eq. (6.56) on page 247.