Introducing the Physics of Low-level Falls

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Background

- Physics is concerned with the most fundamental behavior of the physical world and forms the basis of the other physical sciences and engineering.
- Physics uses the scientific method to formulate and refine theories to explain physical behavior. Predictions must agree with reproducible experimental results within acceptable tolerances.
- The laws of physics are expressed mathematically starting with algebra and symbolic thinking.
- Newton's laws of motion apply universally to all physical bodies in our everyday world.
- Newton's laws were formulated about 300 years ago and have stood the test of time.

Definitions

- **Displacement** ***s*** measures distance between the positions of 2 points. The horizontal distance is represented by Δ*x* and the vertical by Δ*y*, where the Greek letter Δ (*delta*) is traditionally used to designate "the change in" a particular quantity. By definition, Δ*x* ≡ [(final *x*) – (initial *x*)]. The magnitude of the total displacement in 2 dimensions is given by the Pythagorean theorem as $|\boldsymbol{s}| = \sqrt{(\Delta x)^2 + (\Delta y)^2}$.

- **Velocity** ***v*** describes how fast and in what direction an object moves and is usually measured in meters/second (*m/s*) or miles per hour (*mi/hr*), where 1.00 *m/s* = 2.24 *mi/hr*. The average horizontal speed over time interval Δ*t* is defined as $v_{x\ Ave} = \langle v_x \rangle \equiv \Delta x / \Delta t$. The total magnitude is $|\boldsymbol{v}| = \sqrt{{v_x}^2 + {v_y}^2}$. The angular velocity about an axis of rotation, such as the head about the neck, is symbolized as $v_\emptyset$ and is measured in angular *°/s* or *radians/s*, where 1 *radian* or *rad* = 57.3°.

- **Acceleration** ***a*** measures the rate at which the velocity changes, $a_{x\ Ave} = \langle a_x \rangle \equiv \Delta v_x / \Delta t$.

- **Mass** *m* is a measure of the amount of matter. Heavier objects contain more mass than lighter ones. Mass is usually measured in kilograms (*kg*). An object with *m* = 1.00 *kg* will weigh 2.21 pounds on the Earth.

- **Inertia:** An object will remain at rest or in a state of constant uniform motion unless acted upon by an outside force. Force is a push or pull measured in Newtons (*N*) or pounds (*lb*), where 1.00 *N* = 0.225 *lb*. This statement of Newton's 1st law of motion was formulated earlier by Galileo.

- ***a*** = ***F*** / *m***:** Newton's 2nd law of motion states that an object is accelerated in direct proportion to the net applied force, but inversely with the object's mass.

- **Weight =** *mg,* where $g = 9.8\ \frac{m}{s^2} = 32\ \frac{ft}{s^2} = 22\frac{mi/hr}{s}$ is the acceleration due to gravity of all freely-falling objects near the earth's surface.

- **Action = Reaction:** Newton's 3rd law of motion states that the force of interaction between 2 objects must be equal and opposite. This says that if object 1 strikes object 2 with force ***F***, then object 2 will simultaneously strike object 1 with force –***F***. This means that a nail hits the hammer just as hard as the hammer hits the nail, but in the opposite direction.

Falling objects

- A freely falling object moves faster and faster under gravitational acceleration in the vertical direction according to $\Delta v_y = g\Delta t$.

- If the object is initially at rest and falls vertically from height *h*, the resulting impact velocity is $v_{y0} = \sqrt{2gh}$.
(This is valid for short falls while air resistance is negligible.)

- Upon impact with a horizontal surface such as the floor, an upwards force from the surface acts briefly to stop the object and possibly cause rebound.

- The previous force starts from zero at first contact, grows to a maximum or peak value, and finally decreases to zero when contact ceases. The maximum or peak force and resulting acceleration can be twice the average values. These forces are usually much greater that the weight of the object with resulting accelerations much greater than *g.*

Scaling rules

Consider the force resulting from mass *m* falling freely from rest through distance *h* and interacting with a contact surface for time Δt.

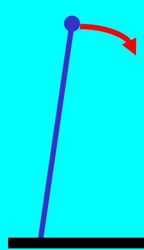
- The force is directly proportional to the mass *m*. This means that twice as much mass results in twice as much force.

- The force is directly proportional to the square root of the height *h*. This means that the height must be increased by 4 times in order to double the force. Consequently, the force is relatively insensitive to changes in height.

- The force is inversely proportional to the contact time Δt. This means that doubling the stopping time will yield half as much force. Conversely, decreasing the stopping time will increase the force accordingly.

Extended objects tipping over

The analysis of extended objects that tip over is more complicated than simple free-fall. However, impact speeds are often comparable.



- Consider a uniform thin rod of length *L* with its center-of-mass at the midpoint that is oriented vertically and then allowed to topple over about an ideal fixed-axis (hinge) at its base onto a flat surface. The eventual impact speed of the top is given by $v_{y0} = \sqrt{3gL}$, which **exceeds** free-fall's $\sqrt{2gL}$ by a factor of $\sqrt{3/2} = 1.22$.

- According to computer simulations, elimination of the above fixed-axis constraint gives even closer agreement with simple free-fall. Reasonable distributions of mass to represent children are used in a series of simulations at this web site to estimate cranial impacts.

Simple physical demonstration

$v_{y0} \approx \sqrt{2gh}$ reasonably approximates the impact speed for a wide variety of cases where the point of eventual contact falls through distance *h* without a fixed axis-of-rotation.

This somewhat surprising result can be demonstrated by dropping and toppling a simple physical model depicted below. It consists of a golf ball tied off in the toe of an ankle-length sock (simulating a head) with the remainder filled with rice (simulating a torso).

Listen to the relative loudness of sounds made by the ball impacting a table under different scenarios. In particular, compare the sound when the model topples over to that when it is aligned horizontally and dropped from the same height.

Rolling off a bed

A straight-forward example of near free-fall occurs when a baby rolls sideways off of a bed, changing table, or any elevation *h*. If the head hits first, its impact velocity is closely approximated by $v_{y0} \approx \sqrt{2gh}$. For a bed that is 0.66 *m* high (26 *inches*), this gives $v_{y0} \approx \sqrt{2(9.8)(0.66)} \approx 3.6$ *m/s* ≈ 8 *mi/hr*. Without a landing cushion to extend contact time Δt and soften the impact with the floor, this is a recipe for disaster.