PHYS 432/532 Formulae for Exam 3 Dr. Hanna

Chapter 1 selected equations:

$$PV = NkT = nRT \quad (1.1 \text{ and } 1.5) \qquad v_{\text{rms}} = \sqrt{\bar{v^2}} = \sqrt{\frac{3kT}{m}} \quad (1.21)$$

$$U_{\text{thermal}} = Nf\frac{kT}{2} \quad (1.23) \qquad \Delta U = Q + W \quad (1.24)$$

$$W = -P\Delta V \to -\int_{V_i}^{V_f} P(V)dV \quad (1.28 \text{ and } 1.29)$$

$$PV^\gamma = \text{const.} \quad (1.40) \qquad \gamma = (f+2)/f$$

$$C \equiv \frac{Q}{\Delta T} \quad (1.41) \qquad c \equiv \frac{C}{m} \quad (1.42)$$

$$C_V = \left(\frac{\Delta U}{\Delta T}\right)_{V,N} \to \left(\frac{\partial U}{\partial T}\right)_{V,N} \quad (1.44) \qquad C_P \to \left(\frac{\partial U}{\partial T}\right)_{P,N} + P\left(\frac{\partial V}{\partial T}\right)_{P,N} \quad (1.45)$$

$$L \equiv \frac{Q}{m} \quad (1.50) \qquad H \equiv U + PV \quad (1.51)$$

$$\Delta H = \Delta U + P\Delta V = Q + W_{\text{other}} \quad (1.53 \text{ and } 1.55) \qquad C_P = \left(\frac{\partial H}{\partial T}\right)_{P,N} \quad (1.56)$$

$$\frac{Q}{\Delta t} \to \frac{dQ}{dt} = -k_t A\frac{dT}{dx} \quad (1.60) \qquad \ell \approx \frac{1}{\sigma n} \approx \frac{1}{4\pi r^2}\frac{V}{N} \quad (1.62)$$

$$\overline{\Delta t} = \frac{\ell}{\bar{v}} \approx \frac{\ell}{v_{\text{rms}}} \quad (1.63) \qquad k_t \approx \frac{1}{2}\frac{C_V}{V}\ell\bar{v} \quad (1.65)$$

$$\frac{|F_x|}{A} = \eta\frac{du_x}{dz} \quad (1.69) \qquad J_x = -D\frac{dn}{dx} \quad (1.70)$$

Chapter 2 selected equations:

$$\Omega(N,n) = \binom{N}{n} \equiv \frac{N!}{n!(N-n)!} \quad (2.6) \qquad \Omega(N,q) = \binom{q+N-1}{q} \equiv \frac{(q+N-1)!}{q!(N-1)!} \quad (2.9)$$

$$N! \approx N^N e^{-N}\sqrt{2\pi N} \quad (2.14) \qquad \ln N! \sim N\ln N - N \quad (2.16)$$

$$\Omega(U,V,N) \approx \frac{1}{N!}\frac{V^N}{h^{3N}}\frac{\pi^{3N/2}}{(3N/2)!}(\sqrt{2mU})^{3N} \quad (2.40) \qquad S = k\ln\Omega \quad (2.45)$$

$$S(U,V,N) = Nk\left\{\ln\left[\frac{V}{N}\left(\frac{4\pi mU}{3Nh^2}\right)^{3/2}\right] + \frac{5}{2}\right\} \quad (2.49)$$

Chapter 3 selected equations:

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{V,N} \quad (3.5) \qquad (dS)_{V,N} = \frac{dU}{T} = \frac{Q}{T} = \frac{C_V dT}{T} \quad (\text{const. } V, \text{ no work}) \quad (3.17 \text{ and } 3.18)$$

$$U = -MB = -N\mu B\tanh\left(\frac{\mu B}{kT}\right) \quad (3.31 \text{ and } 3.32) \qquad C \to 0 \quad \text{as} \quad T \to 0 \quad (3.22)$$

$$\frac{P}{T} = \left(\frac{\partial S}{\partial V}\right)_{U,N} \quad (3.39) \qquad Q = TdS \quad (\text{quasistatic}) \quad (3.48)$$

$$-\frac{\mu}{T} = \left(\frac{\partial S}{\partial N}\right)_{U,V} \quad (3.55) \qquad dU = TdS - PdV + \mu dN \quad (3.58)$$

Chapter 4 selected equations:

$$e \equiv \frac{\text{benefit}}{\text{cost}} = \frac{W}{Q_h} \quad (4.1) \qquad e \le 1 - \frac{T_c}{T_h} \quad (4.5)$$

$$\text{COP} = \frac{\text{benefit}}{\text{cost}} = \frac{Q_c}{W} \quad (4.6) \qquad \text{COP} \le \frac{1}{T_h/T_c - 1} = \frac{T_c}{T_h - T_c} \quad (4.9)$$

$$H = U_{\text{potential}} + U_{\text{kinetic}} + PV \quad (4.19)$$

Chapter 5 selected equations:

$$F = U - TS \quad (5.2) \qquad G = U - TS + PV \quad (5.3)$$

$$\Delta F \le W \quad \text{(at constant } T, V, N) \quad (5.5) \qquad \Delta G \le W_{\text{other}} \quad \text{(at constant } T, P, N) \quad (5.8)$$

Obtain Eqs. (5.18), (5.20), and (5.23) for dH, dF, and dG from Eq. (3.58) for dU, and the definitions of H, F, and G in Eqs. (1.51), (5.2), and (5.3).

$$G = N\mu \quad (5.35) \qquad G = N_1\mu_1 + N_2\mu_2 + \cdots = \sum_i N_i\mu_i \quad (5.37)$$

$$\mu\left(T, P\right) = \mu^\circ\left(T\right) + kT\ln\left(P/P^\circ\right) \quad (5.40)$$

$$\frac{dP}{dT} = \frac{S_g - S_l}{V_g - V_l} = \frac{mL}{T\Delta V} \quad (5.46 \text{ and } 5.47) \quad \ldots \text{ assumes Eq. (1.50) for } L$$

$$\left(P + \frac{aN^2}{V^2}\right)(V - Nb) = NkT \quad (5.49)$$

$$G = (1-x)G_A^\circ + xG_B^\circ \quad \text{(unmixed)} \quad (5.59)$$

$$\Delta S_{\text{mixing}} = -nR[x\ln x + (1-x)\ln(1-x)] \quad (5.60)$$

$$G = (1-x)G_A^\circ + xG_B^\circ + nRT\left[x\ln x + (1-x)\ln(1-x)\right] \quad \text{(ideal mixture)} \quad (5.61)$$

Chapter 6 selected equations:

$$\mathcal{P}(s) = \frac{1}{Z}e^{-E(s)/kT} \quad (6.8) \qquad Z = \sum_s e^{-E(s)/kT} \quad (6.10)$$

$$\overline{X} = \sum_s X(s)\mathcal{P}(s) = \frac{1}{Z}\sum_s X(s)e^{-\beta E(s)} \quad (6.18) \qquad \overline{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} \quad (6.25)$$

$$\mathcal{D}(v) = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 e^{-mv^2/2kT} \quad (6.50) \qquad F = -kT\ln Z \quad (6.56)$$

$$Z_{\text{total}} = Z_1 Z_2 Z_3 \cdots Z_N \quad \text{(noninteracting, distinguishable systems)} \quad (6.69)$$

$$Z_{\text{total}} = \frac{1}{N!}Z_1^N \quad \text{(noninteracting, indistinguishable particles)} \quad (6.70)$$

$$\ell_Q = \frac{h}{\sqrt{2\pi mkT}} \quad (6.80) \qquad v_Q = \ell_Q^3 = \left(\frac{h}{\sqrt{2\pi mkT}}\right)^3 \quad (6.83)$$

$$Z_{\text{tr}} = \sum_s e^{-E_{\text{tr}}/kT} = \sum_{n_x}\sum_{n_y}\sum_{n_z} e^{-h^2 n_x^2/8mL_x^2 kT} e^{-h^2 n_y^2/8mL_y^2 kT} e^{-h^2 n_z^2/8mL_z^2 kT}$$

$$\rightarrow \frac{L_x}{\ell_Q}\frac{L_y}{\ell_Q}\frac{L_z}{\ell_Q} = \frac{V}{v_Q} \quad (6.82)$$

$$Z = \frac{1}{N!}\left(\frac{VZ_{\text{int}}}{v_Q}\right)^N \quad (6.85) \qquad \mu = \left(\frac{\partial F}{\partial N}\right)_{T,V} = -kT\ln\left(\frac{VZ_{\text{int}}}{Nv_Q}\right) \quad (6.93)$$

Chapter 7 selected equations:

$$\mathcal{P}(s) = \frac{1}{\mathcal{Z}} e^{-[E(s)-\mu N(s)]/kT} \quad (7.6) \qquad \mathcal{Z} = \sum_s e^{-[E(s)-\mu N(s)]/kT} \quad (7.7)$$

$$\overline{n}_{\rm FD} = \frac{1}{e^{(\epsilon-\mu)/kT}+1} \quad (7.23) \qquad \overline{n}_{\rm BE} = \frac{1}{e^{(\epsilon-\mu)/kT}-1} \quad (7.28)$$

$$\epsilon_{\rm F} \equiv \mu(T=0) \quad (7.33) \qquad \epsilon_{\rm F} = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3} \quad (7.39)$$

$$U = \frac{3}{5}N\epsilon_{\rm F} + \frac{\pi^2}{4}N\frac{(kT)^2}{\epsilon_{\rm F}} \quad (7.47) \qquad g(\epsilon) = \frac{\pi(8m)^{3/2}}{2h^3}V\sqrt{\epsilon} = \frac{3N}{2\epsilon_{\rm F}^{3/2}}\sqrt{\epsilon} \quad (7.51)$$

$$\overline{n}_{\rm Pl} = \frac{1}{e^{hf/kT}-1} \quad (7.72) \qquad u(\epsilon) = \frac{8\pi}{(hc)^3}\frac{\epsilon^3}{e^{\epsilon/kT}-1} \quad (7.84)$$

$$\frac{U}{V} = \frac{8\pi^5(kT)^4}{15(hc)^3} \quad (7.86) \qquad \frac{\text{power}}{\text{area}} = \frac{2\pi^5}{15}\frac{(kT)^4}{h^3c^2} = \sigma T^4 \quad (7.97)$$

$$U = \frac{9NkT^4}{T_{\rm D}^3}\int_0^{T_{\rm D}/T}\frac{x^3}{e^x-1}dx \quad (7.112)$$

$$kT_c = 0.527\left(\frac{h^2}{2\pi m}\right)\left(\frac{N}{V}\right)^{2/3} \quad (7.126) \qquad N_{\rm excited} = \left(\frac{T}{T_c}\right)^{3/2}N \quad (7.128)$$

Constants and units:

$k = 1.381\times10^{-23}$ J/K $= 8.617\times10^{-5}$ eV/K, $N_{\rm A} = 6.022\times10^{23}$, $R = N_{\rm A}k = 8.315$ J/mol·K

$h = 6.626\times10^{-34}$ J·s, $c = 2.998\times10^{8}$ m/s, $e = 1.602\times10^{-19}$ C

$m_e = 9.109\times10^{-31}$ kg, $m_p = 1.673\times10^{-27}$ kg

Temperature scales: 0 °C = 32 °F = 273.15 K, 100 °C = 212 °F

Pressure: 1 atm = 1.013 bar = 1.013×10^5 Pa = 14.7 lb/in^2 = 760 mm Hg

Water: $c = 1$ cal/g°C = 4186 J/kg°C, $\rho = 1$ g/cm^3 = 1000 kg/m^3,

$L_{\rm melt} = 333$ J/g @ 1 atm, $L_{\rm boil} = 2260$ J/g @ 1 atm

Stefan-Boltzmann constant: $\sigma = \frac{2\pi^5k^4}{15h^3c^2} = 5.67\times10^{-8}$ W/m^{2}K^4