BEC Vortex Matter

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October 6, 2006

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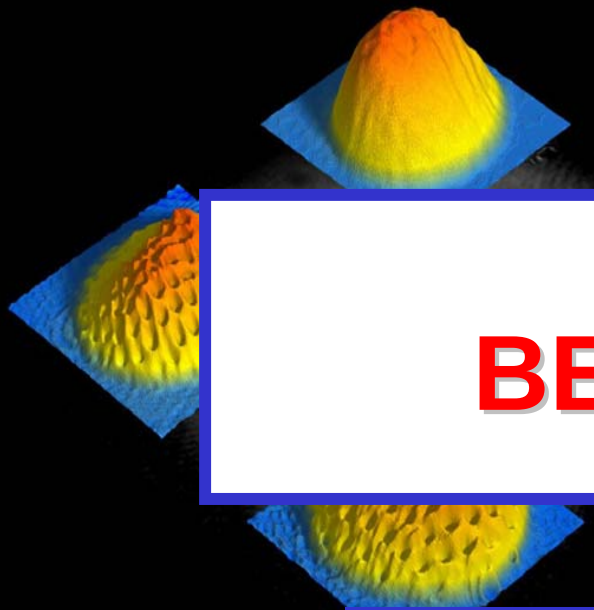
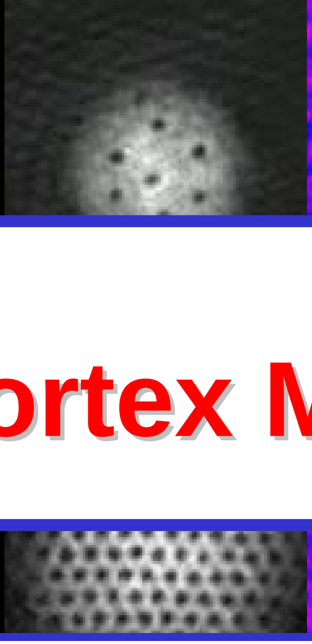
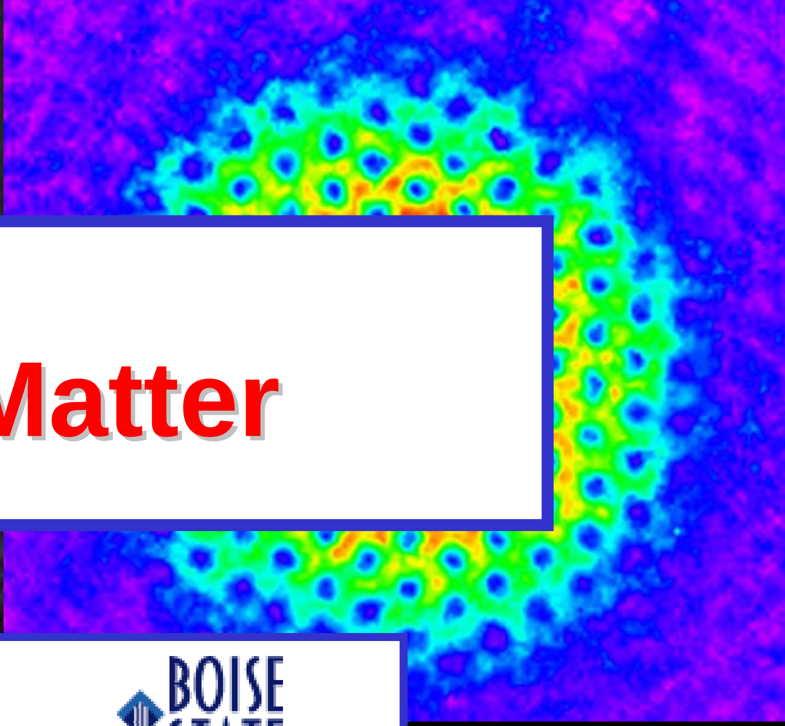
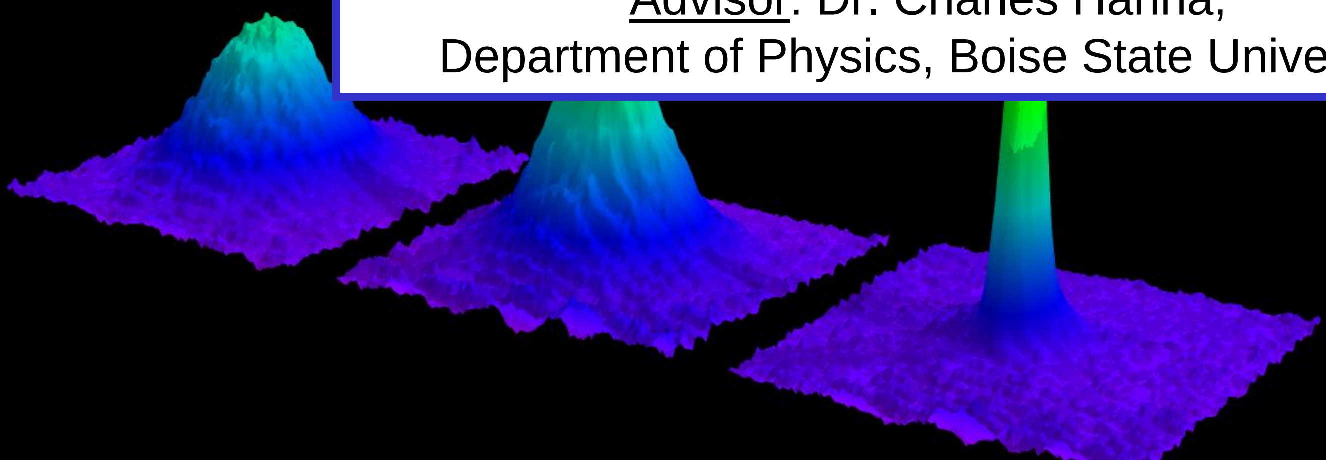
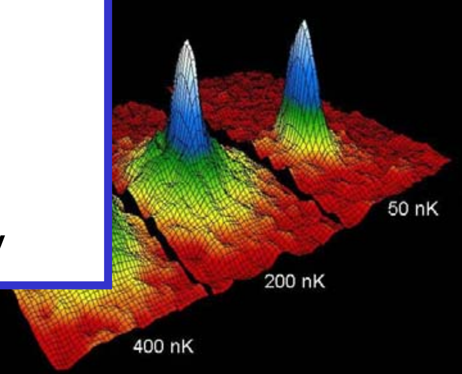
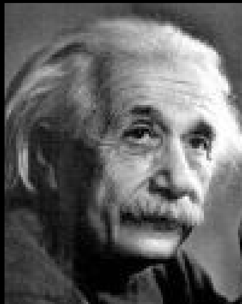








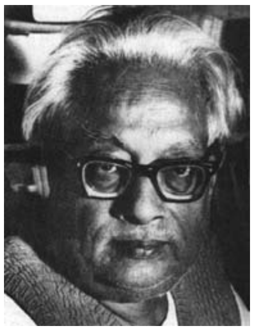




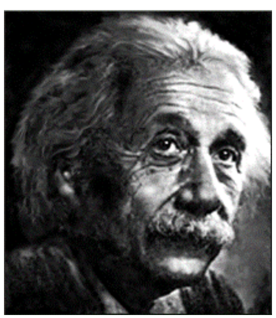
Outline

1. Bosons: what are they?
2. Bose-Einstein Condensation (BEC)
3. Vortex Formation: Rotating BECs
4. Current Work: theory, visualization, results
5. Acknowledgements

History of Bosons



S. Bose



A. Einstein

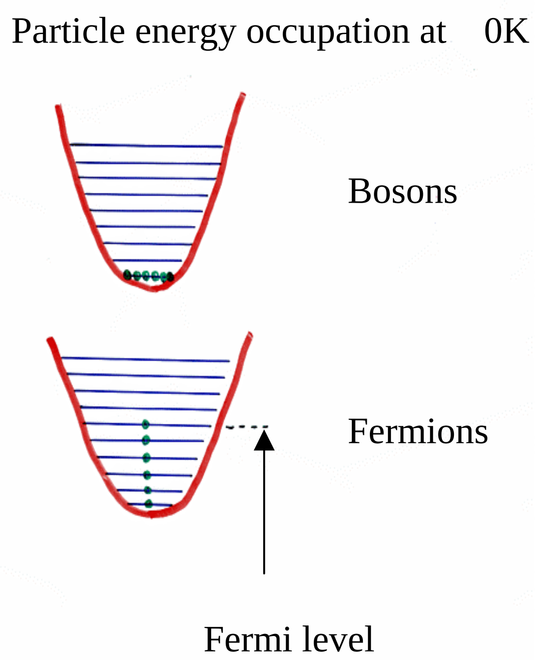
- 1924 – Physicist S. Bose realized that classical particle statistics was insufficient for describing photons and developed a new statistics for photons

$$f(E) = \frac{1}{e^{E/kT}} \longrightarrow f(E) = \frac{1}{e^{E/kT} - 1}$$

Boltzmann distribution — becomes — Bose-Einstein distribution!

- 1924 – A. Einstein generalized this formulation to massive particles (i.e., matter)

What is a Boson?



W. Ketterle, *A new form of matter: Bose-Einstein condensation and the atom laser*, 1998

- Bosons:
  - examples: photons, ^{4}He, ^{87}Rb
  - integer spin (0,1,2,…)
  - prefer to be in the same quantum state

  …satisfy Bose-Einstein statistics

- Fermions:
  - examples: electrons, protons, neutrons
  - fractional spin (1/2,3/2,…)
  - cannot be in the same quantum state (Pauli exclusion principle)

  …satisfy Fermi-Dirac statistics

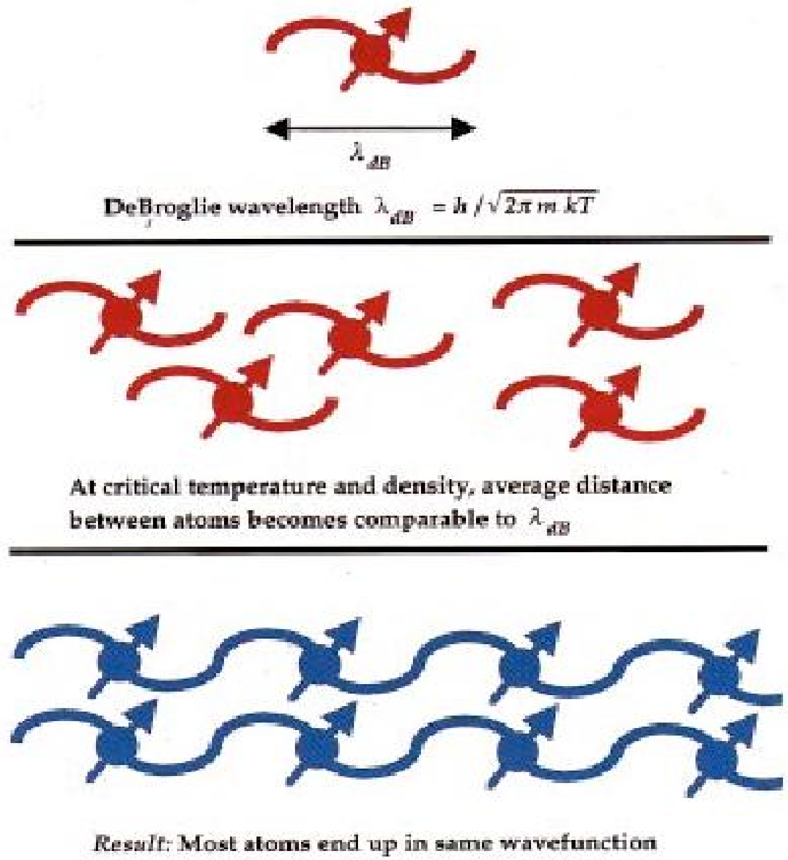
What is a BEC?

"From a certain temperature on, the molecules condense without attractive forces, that is, they accumulate at zero velocity. The theory is pretty but is there also some truth to it ?" - Albert Einstein

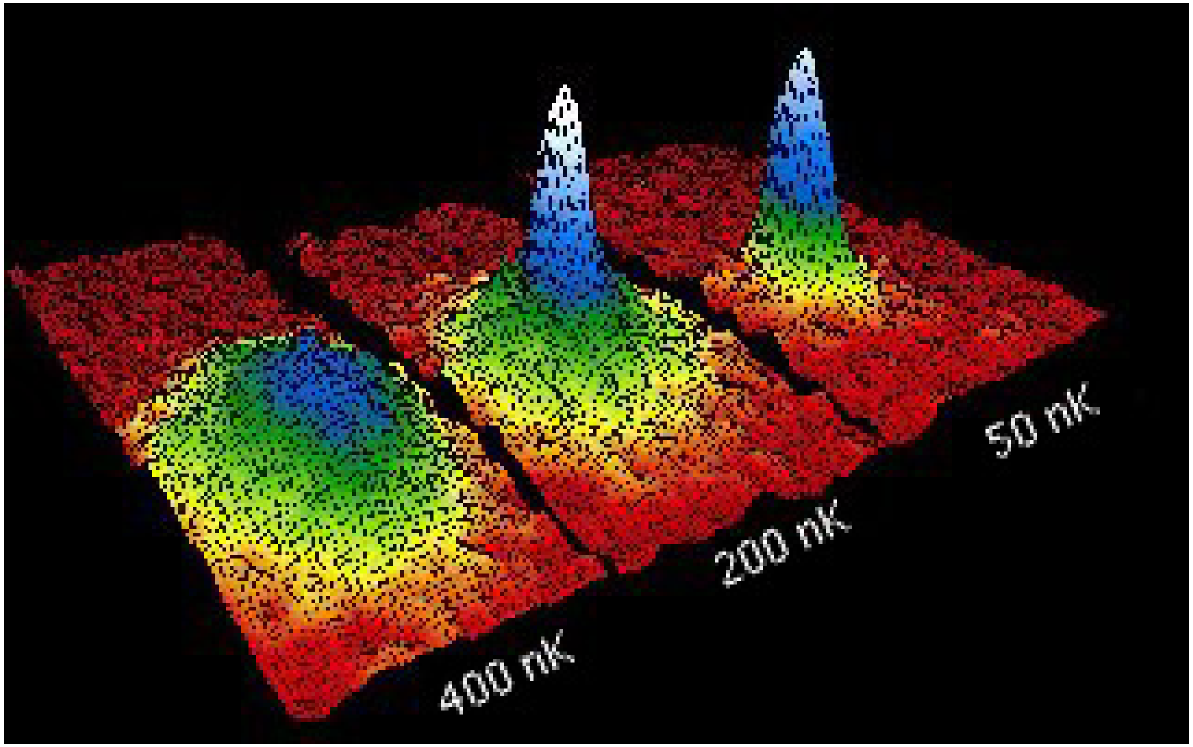
- Matter waves have an associated de Broglie wavelength

$$\lambda_{dB} = h/p$$

- As matter cools, λ_{dB} grows
- When $\lambda_{dB} \sim r_0$, most atoms have same wave function …BEC!



Is It Condensed?



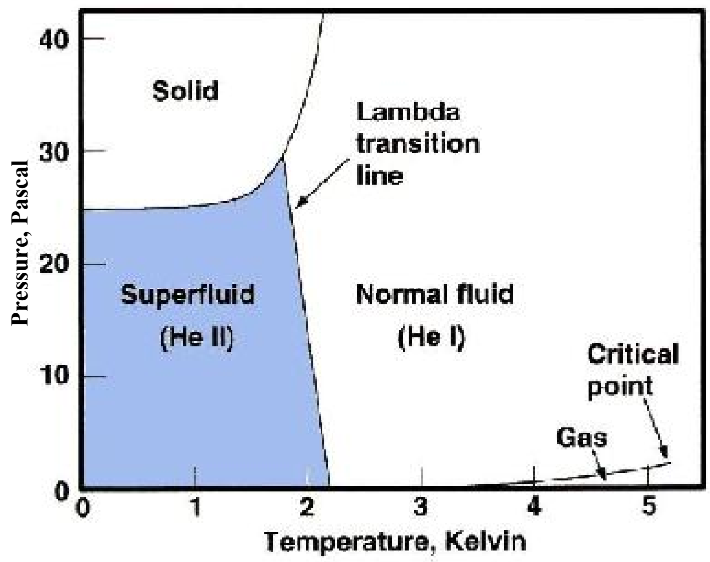
www.maloka.org/f2000/bec/ what_it_looks_like.html

- Velocity distribution as a dilute gas is cooled into the nano-Kelvin range.
- Distribution shows particles accumulate at zero velocity.
- Experimental proof of BEC!

History of Bose-Einstein Condensates (BECs)

- 1924 – Bose and Einstein describe statistics of bosons
- 1932 – Keesom and Clusius measure the "λ-point" temperature of liquid ^4He. Below this temperature, ^4He becomes a superfluid.
- 1938 – London suggested an explanation for the properties of cold ^4He: treat as an ideal gas obeying Bose-Einstein statistics (a cold aggregate of bosons in their lowest energy level).
- Today – Similar aggregates of cold bosons are studied using dilute atomic gases such as ^{87}Rb, ^7Li, and ^{23}Na.

Phase diagram for ^4He



R.Rothe, *Bose-Einstein Condensation: A new kind of matter*, 1995

History of BECs:
The pioneers of modern BECs

Key steps to BEC with a cold gas of atoms:

1997 

Chu, Cohen, Phillips

- 1^{st} The development of the laser cooling process

http://www.colorado.edu/physics/2000/bec/index.html

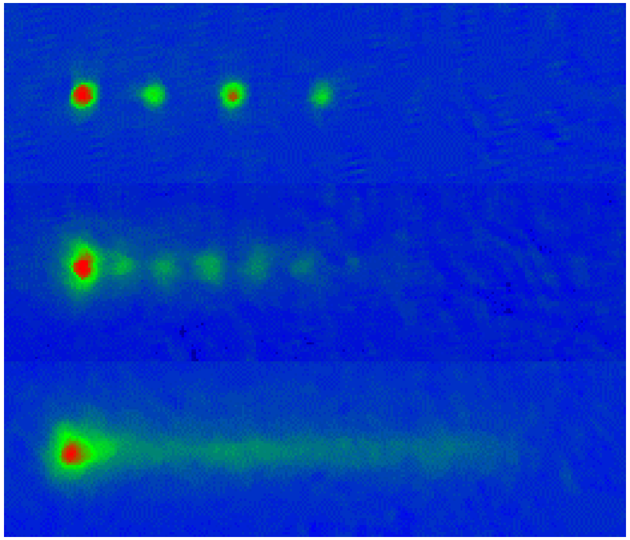
2001 

Cornell, Ketterle, Weiman

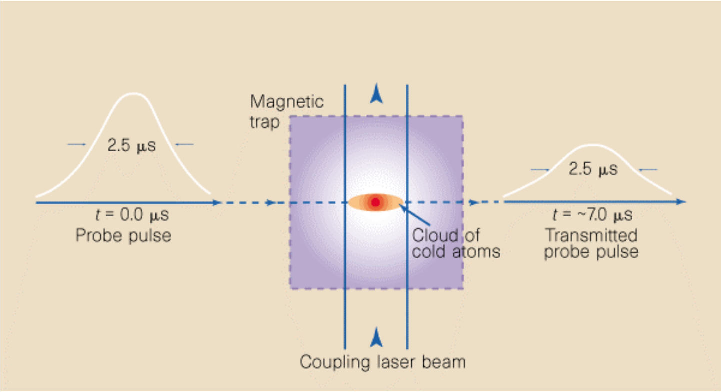
- 2^{nd} The creation of BECs in dilute gases of alkali atoms

Potential Application of BECs

- Atom Lasers
  - Quantum Lithography
  - Matter-Wave Inferometry
- Ultra precise gyroscopes
  - potentially 10^{10} times more accurate than laser gyroscopes



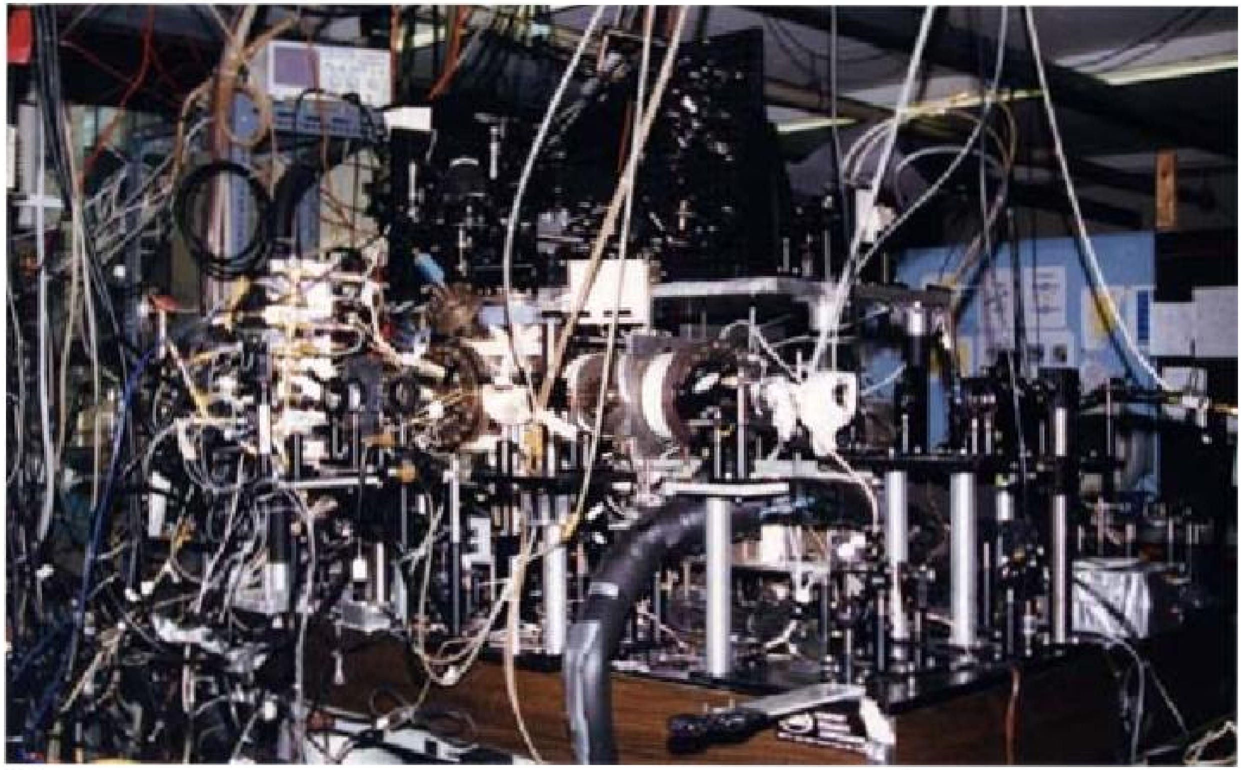
- Slow Light
  - Information Storage, Manipulation
- Quantum Computation?
  - BECs in optical lattices



Nature © Macmillan Publishers Ltd 1999

BEC apparatus

- Condensing dilute atomic gases at MIT: BECs are now a reality.

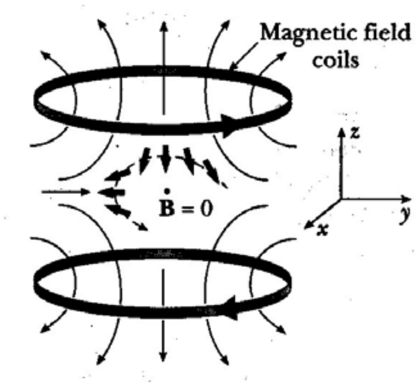


^{23}Na setup @ MIT; *W. Ketterle, et al.*

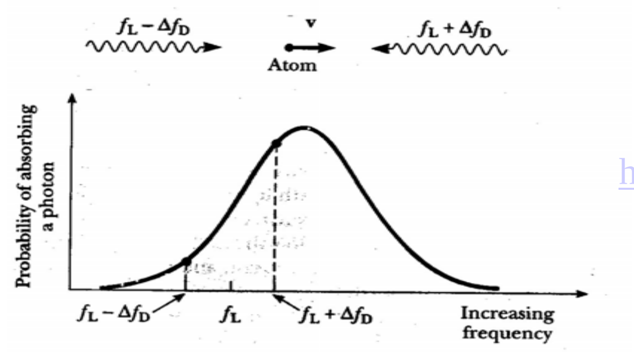
Trapping and Cooling the Substrate

- Magnetic Trap

http://www.colorado.edu/physics/2000/bec/mag_trap.html

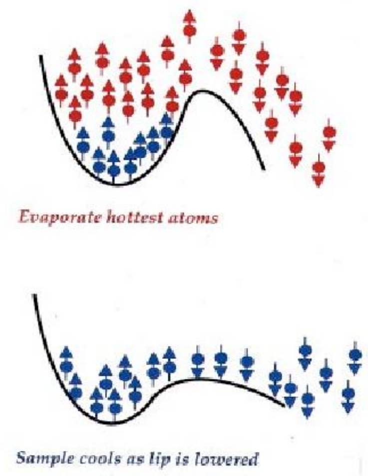


Serway, Moses, Moyer, *Modern Physics* Saunders College Publishing, 1997



- Evaporative Cooling

http://www.colorado.edu/physics/2000/bec/evap_cool.html



Cornell, *Very Cold Indeed: The nanokelvin physics of Bose-Einstein condensates*, 1996

- Laser Cooling

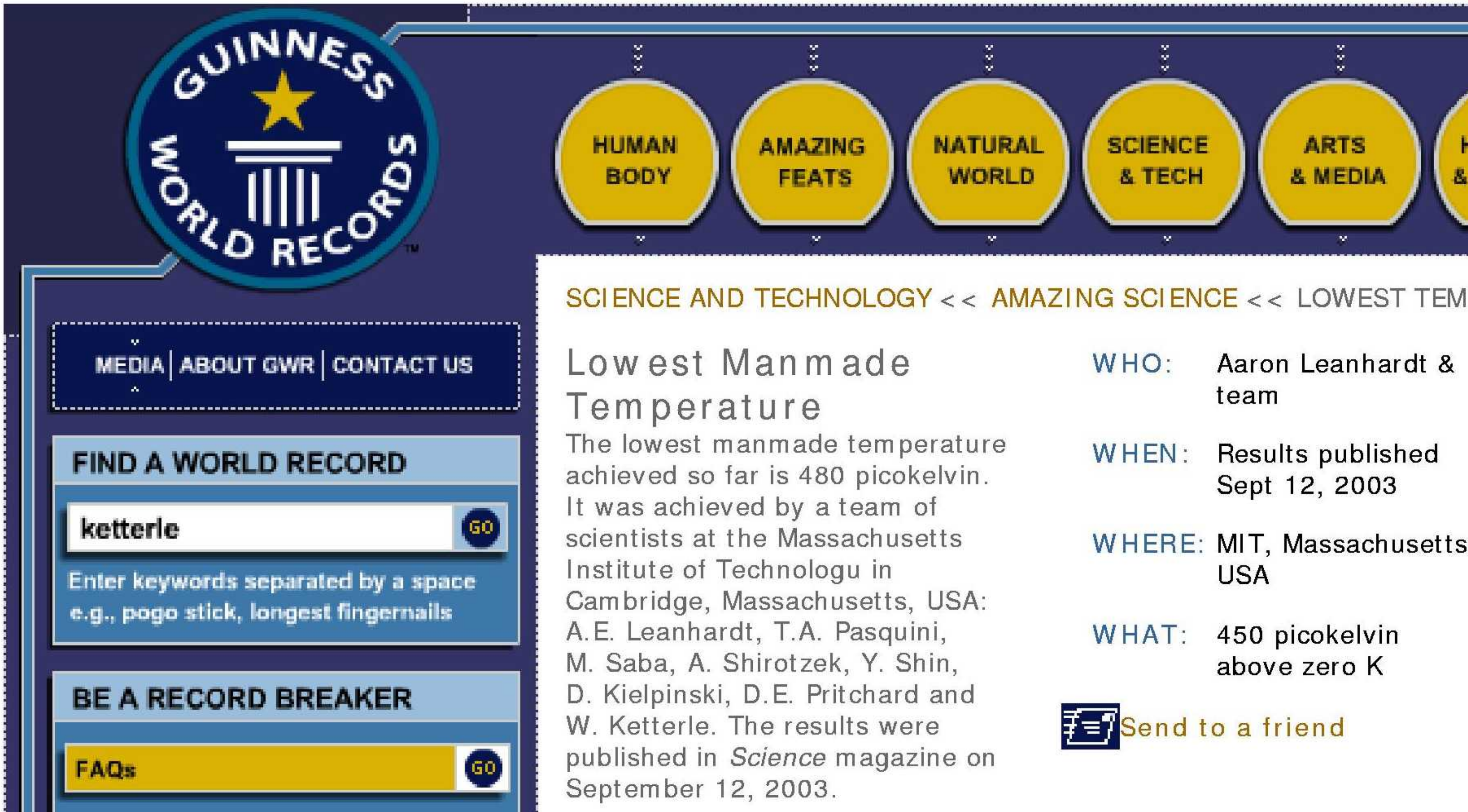
http://www.colorado.edu/physics/2000/bec/lascool1.html

Serway, Moses, Moyer, *Modern Physics* Saunders College Publishing, 1997

Guinness World Record:

$T < 0.5\,\mathrm{nK}$

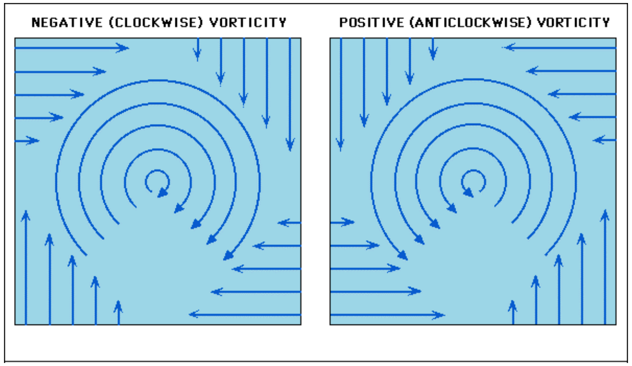
Ketterle *et al.*



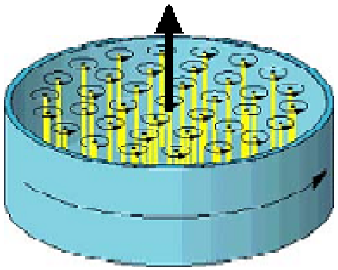
Rotating BECs:
Vorticity

- Strange property of BEC superfluids: they are irrotational.
- 1955 – R. Feynman proposes that in *rotating* superfluids, quantized regions of vorticity appear.
- Large aggregates of vortices appear in the fluid, if we spin the BEC rapidly enough.

Spinning BECs ⟶ Vortex Matter!



Vorticity in fluid flow



http://www.ldeo.columbia.edu/edu/dees/ees/climate/slides/vorticity.gif

How do you spin BECs?

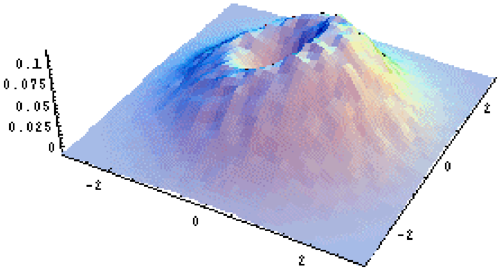
With Lasers!

- Lasers are shone on the BEC, which knocks bosons out of the condensate.
- The radius of the condensate decreases, and the mass decreases.
- If the radius and mass of a spinning object decrease, that object will spin faster due to conservation of momentum:

$$L = m \cdot r \cdot \omega$$

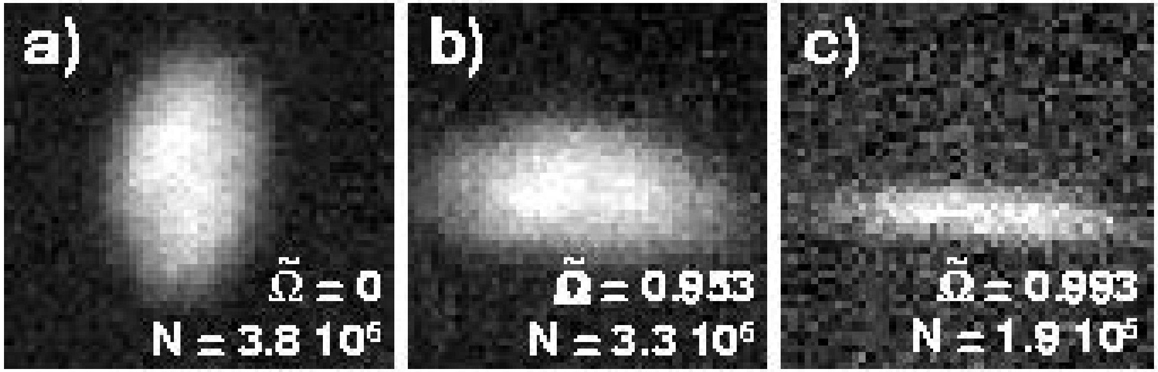
And with spinning traps!

- BEC is distorted, then the MOT is set to spin.
- Spinning MOT induces spinning BEC.



Spinning BECs:
2D Condensates

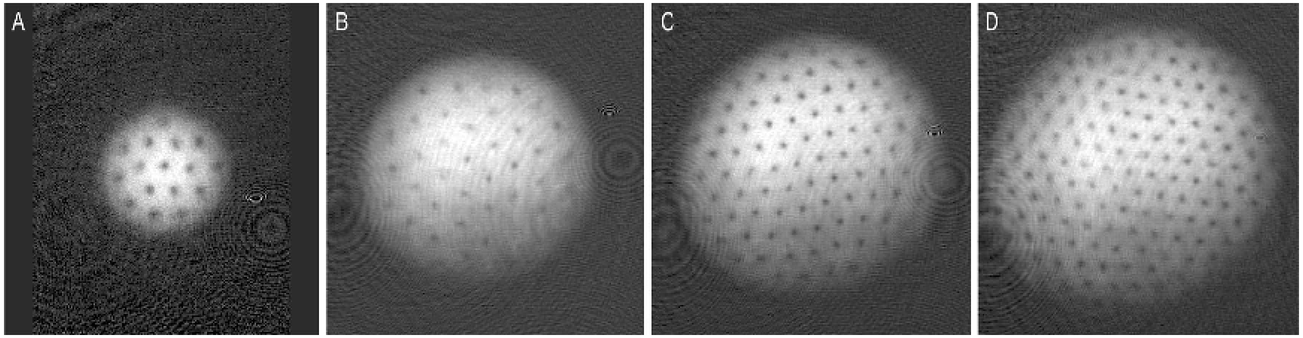
- As the BEC spins, it begins to “flatten out” due to centrifugal forces
- The higher the rotation rate becomes, the flatter the BEC gets
- Approximate our BEC as a 2D system $\quad gN << \hbar\Omega_z$



V. Schweikhard, I. Coddington, P. Engels, V.P. Mogendorff, E.A. Cornell, *Rapidly Rotating Bose-Einstein Condensates in and near the Lowest Landau Level*. e-print arXiv:cond-mat/0308582, Aug 27, 2003.

Vortices:
Stabilizing the Lattice

Vortex Matter!



cua.mit.edu/.../Projects_2001/ Vortex_lattice/Vortex.htm

- As the rate of spinning increases, the vortices stabilize, forming vortex arrays
- Triangular Abrikosov lattice
- Hundreds of vortices

Materials Science of Vortex Matter

- Unusual form of matter
  - Macroscopic quantum phenomena
  - Obeys Eulerian dynamics (moves perpendicular to $\vec{F}$)

- Areas of Study
  - Phases, Statics, Dynamics, Mean-Field theory
  - Quantum fluctuations, quantum melting, exotic states

- Comparison to experiment…(e.g. group of E.A. Cornell)

Rotating BEC's – H_{eff}

How to treat a rotating system?:
go to the rotating frame

$$H_\Omega = H_{\text{lab}} - \mathbf{\Omega} \cdot \mathbf{L}$$

$$H_\Omega^0 = \frac{(\mathbf{p} - m\Omega\hat{z} \times \mathbf{r})^2}{2m} + \frac{1}{2}m(\Omega_R^2 - \Omega^2)(x^2 + y^2) + \frac{1}{2}m\Omega_Z^2 z^2$$

B_{eff} in z-dir with $\Omega_c = 2\,\Omega$

reduced radial confinement

Same as charged particles in a strong magnetic field!

Rapidly rotating limit $\Omega \rightarrow \Omega_R$

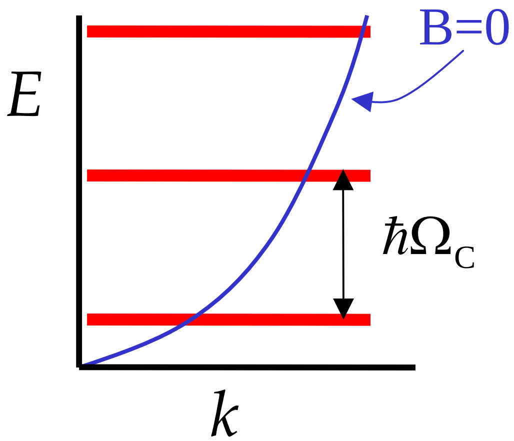
Landau Levels:
2D particles in a strong B field

Classical: cyclotron frequency

$$\Omega_C = \frac{eB}{m}$$

Quantum Mechanical:
Landau levels

$$E_n = \hbar\Omega_C (n + \frac{1}{2})$$



Landau levels are macroscopically degenerate

n=0 LLL $gN << \hbar\Omega_C$ **Lowest Landau Level approx.**

$$z = x + iy$$

$$\phi(z) = \Phi_0 e^{-|z|^2/4} \prod_{i=1}^{N_V} (z - z_i)$$

Vortex Matter:
Statics

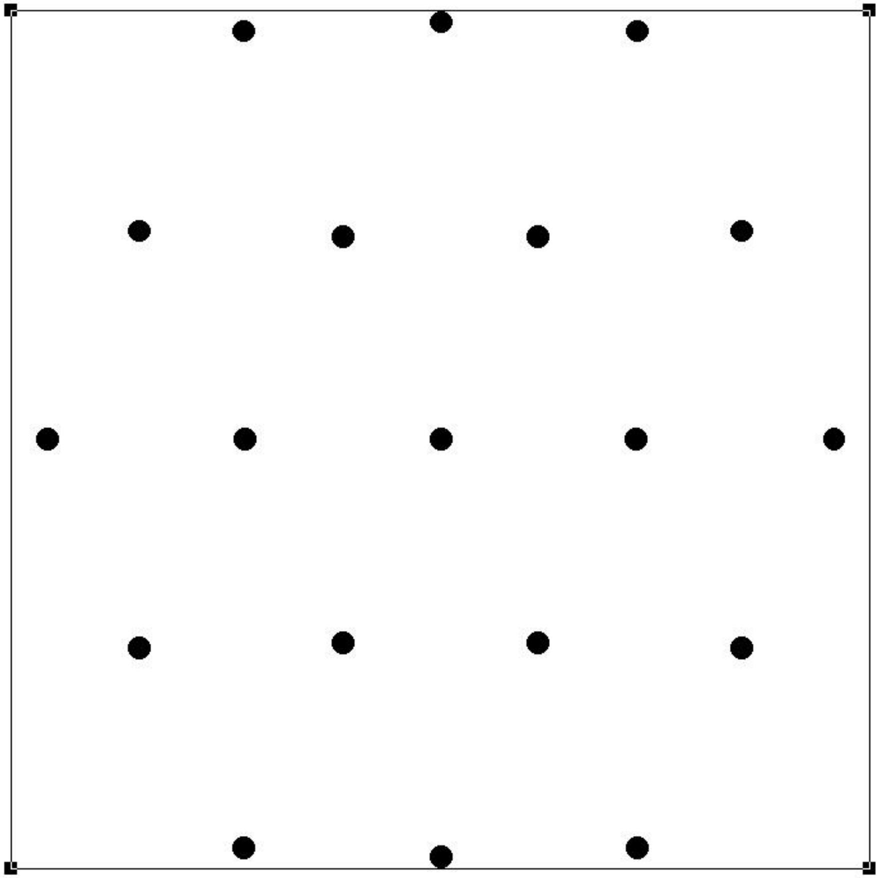
- Equilibrium: $\vec{F}_{net} = 0$

$$\frac{E}{N} = \int d^2 z \left[V(z) |\phi(z)|^2 + \frac{gN}{2} |\phi(z)|^4 \right]$$

Trapping Term — Interaction Term

$$f_i = -\frac{\partial}{\partial z_i^*}\left(\frac{E}{N}\right) = 0$$

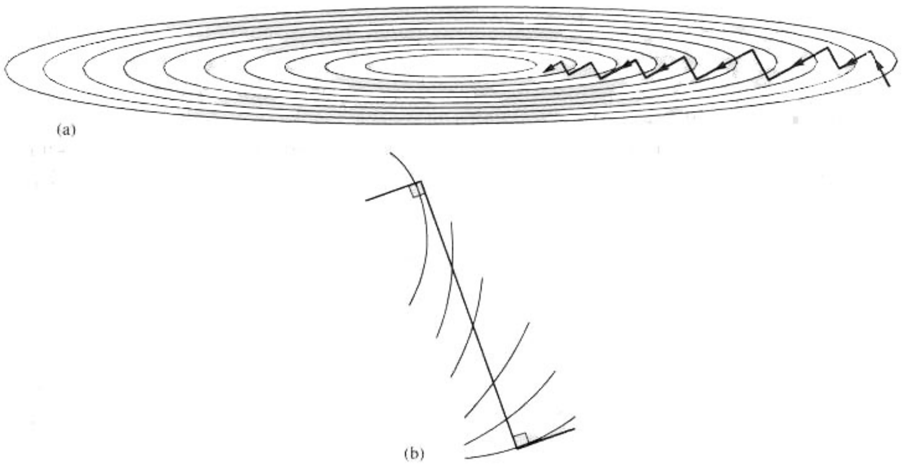
At equilibrium, the force at each vortex is zero.



Numerical Energy Minimization

- Steepest Descent
  - Robust, yet inefficient method
  - Step directions perpendicular to previous step

$$d_i \bullet d_j = 0$$

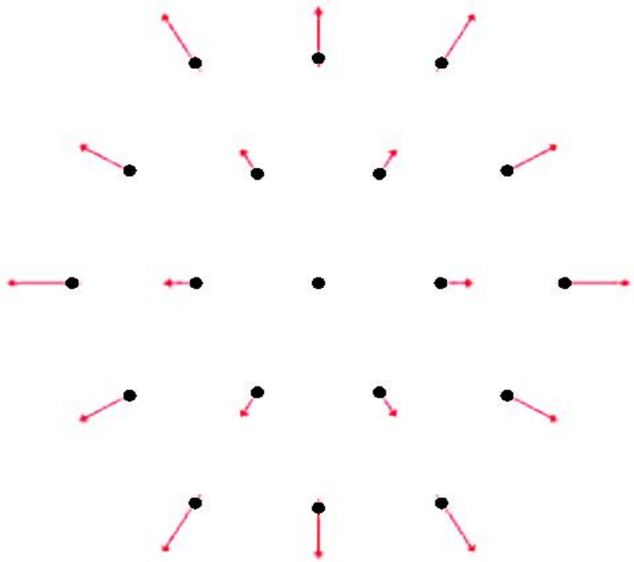


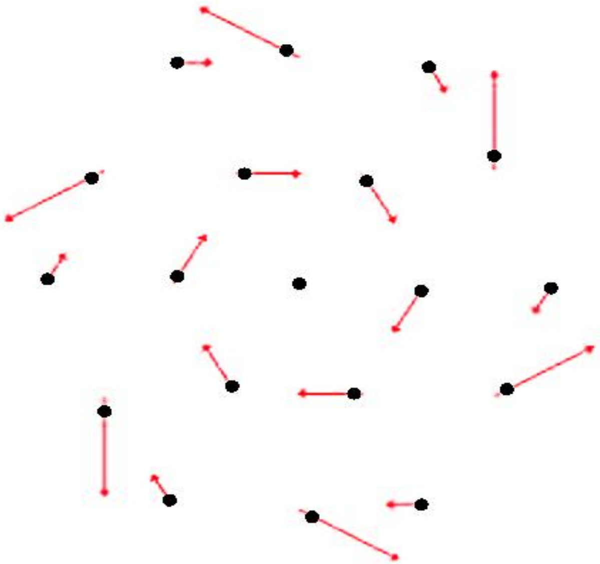
- Conjugate Gradient
  - Step directions conjugate to previous step
  - Efficiently reaches local extremum in $O(N_V)$ steps

$$d_i \cdot A \cdot d_j = 0$$

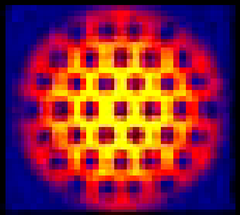
Vortex Matter:
Dynamics

- Second derivatives of the energy determine the restoring forces
- Action diagonalized using the Bogoliubov transformation
- Analogous to vortices connected by springs
- Eulerian dynamics: motion perpendicular to applied force
- For N_v vortices, there are many numerically distinct physical modes of oscillation





eigenvalue(1): 0.000000



Vortex Dynamics: The Movie*

Starring: 37 Vortices in the Lowest Landau Level

Director & Producer: J. C. Diaz-Velez, Boise State Univ.
Executive Producer: C. B. Hanna, Boise State University

Based on calculations by
C. B. Hanna, Boise State University
Jairo Sinova, Texas A&M
A. H. MacDonald, University of Texas at Austin

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the National Science Foundation under Grant Nos.
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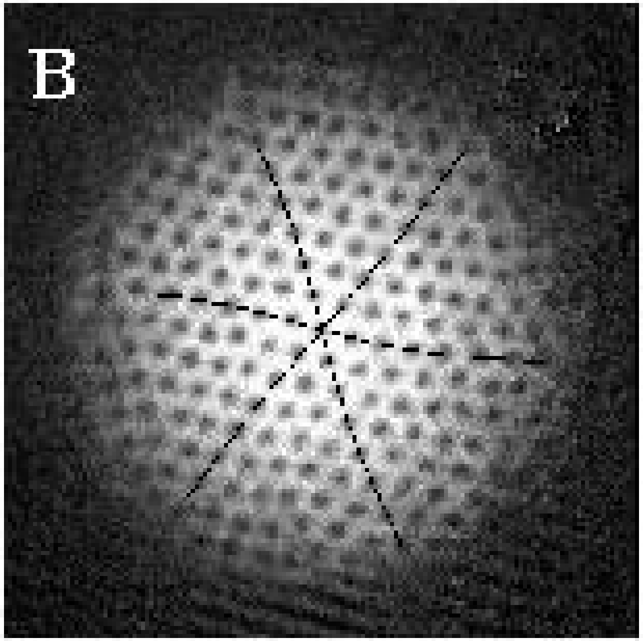
Simulating vortex matter:
An interactive approach via applets

- Meaningful comparisons to experiment can be made via an interactive simulation.
- Java applets are a convenient, interactive solution.

http://newton.boisestate.edu/~asup/VortexApplet.html



E.A.Cornell, et.al., *Observation of Tkachenko Oscillations in Rapidly Rotating Bose-Einstein Condensates*, arXiv:cond-mat/0305008, 2003

Acknowledgements

- Dr. Charles Hanna
  - Department of Physics, Boise State University
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  - Department of Mathematics, Boise State University

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