

MATHEMATICS INSTRUCTION

Word Problem Solving Rubric Manual

Angela R. Crawford, Evelyn S. Johnson, Laura A. Moylan, and Yuzhu Zheng

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Word Problem Solving Rubric

Overview:

The Word Problem Solving rubric (WPS) was designed for use by supervisors and administrators to reliably evaluate teachers' implementation of practices that support students' abilities to solve word problems. Word problems are tasks that involve mathematical information presented as text rather than in symbolic notation, usually presented in story situations but which may include other presentations such as charts and graphs. Problem solving has been defined as the process of detecting the salient information and processes necessary for translating between the given task and the answer (Goldin, 1982; Xin et al., 2011).

This rubric reflects conceptual model-based, or schema-based, problem solving instruction for solving word problems. Research indicates that schema-based instruction is more effective for students with learning difficulties than instruction based on general heuristic, keyword strategies, or metacognitive instruction only (see for example, Fuchs et al., 2003; Jitendra, Griffin, Deatline-Buchman, & Sczesniak, 2007; Xin & Jitendra, 1999; Xin et al., 2011). Schema-based instruction includes modeling problems as schematic representations, scaffolded instruction, and strategy training.

Schema(ta): A schema (schemata, plural) is a conceptual model of the underlying mathematical structure of a problem. Schemata are generalizable conceptual models that apply to many variations of problem contexts and determine the development of a solution plan. It is important during instruction that word problems are presented that have missing values (unknown quantity) in any position in the schema. This results in three variations for each problem type.

Schemata can be organized as followed:

--**part-part-whole** (PPW): part + part = whole, generalizable conceptual model for addition or subtraction. There are three types of PPW schemata:

--change schema-join or separate (I had 10 pencils. I lost/gained 4 pencils. Now I have ... pencils.)

--combine schema (I have 6 pencils. You have 4 pencils. Together we have 10 pencils.)

--**additive compare** (AC): larger quantity - smaller quantity = difference; generalizable conceptual model for addition and subtraction. There are two types of AC schemata:

--compare-more schema (I have 6 pencils. You have 4 more pencils. You have 10 pencils.)

--compare-less schema (I have 6 pencils. You have 4 fewer pencils. You have 2 pencils.)

--**factor-factor-product** (FFP): factor x factor = product, generalizable conceptual model for multiplication and division. There are two types of FFP schemata:

--equal groups schema (unit rate x # of units = product/size of groups x number of groups = total; I have 4 pencil boxes. Each box has 6 pencils. There are 24 pencils in all.)

--multiplicative compare (referent unit x multiplier = compared/product; I have 6 pencils. You have 4 times more pencils. You have 24 pencils.)

--**ratio/proportion** (R/P): generalizable conceptual model for rates and proportions.

There are five types of R/P schemata:

--ratio (compared/base = ratio value; 6 students have dogs/4 students do not have dogs. The ratio is 6:4 or 3:2.

--percent (part-compared/whole-base = ratio value x/100)

--simple percent of change (change-compared/original-base = ratio value x/100)

--complex percent of change (two step: change-compared/original-base = ratio value x/100; original & change = new)

--simple interest (two step: change-compared/original-base = ratio value x/100; original/principal & change/interest = new)

Schematic representation: Representation is the activity of translating mathematical ideas from one form into another. In this rubric, *representations* refer to translations that are presented in a visual, schematic form. The schematic representations, or diagrams, are the conceptual model that drives a solution plan or accurate problem solving. These diagrams reflect the underlying structure or mathematical relationships in the problem rather than surface features (such as the particular people or objects involved).

Schema instruction phase: Instruction is scaffolded by beginning with conceptual understanding of the schema with no unknown quantities. Conceptual understanding of the underlying structure and the ability to represent it in a schematic diagram are emphasized before instruction that focuses on solving for unknown quantities. There is a gradual release of responsibility for representations from teacher to students, and this may include providing students with cue cards or other guides for correctly identifying the components of the schema.

Problem solving phase: Once students have demonstrated understanding of the particular schema, instruction turns to solution planning for problems with unknowns and accurate problem solving, again with gradual release of problem solving from teacher to student. This phase of instruction may include strategy training such as heuristics or checklists that guide students through the steps of the problem solving process.

In both phases of instruction, simple number sets precede more difficult number sets, and familiar contexts precede more unfamiliar or complex contexts. In the second phase, solution planning and problem solving, the location of the missing number is varied.

Only one new schema is introduced at a time, previously taught schemata are then contrasted, and two-step problem solving instruction is based on previously taught schemata.

Overall, instruction involves explicit explanation and modeling, active student involvement, guided practice, independent practice, and monitoring and specific feedback.

The rubric provides specific, accurate, and actionable feedback to special education teachers about the quality of their instruction, and ultimately, improve the outcomes for students with disabilities. The purpose of this manual is to provide technical information for implementing the WPS rubric as a tool for evaluation and feedback.

This rubric includes 5 components. These are:

- 1) Content/Design of Instruction
- 2) Sequence of Instruction
- 3) Delivery of Instruction
- 4) Student Engagement
- 5) Providing Feedback

Under these 5 components, there are 17 items. For each item, there are five levels of implementation. Observing either live or from video, the observer assigns a rating based on a scale that ranges from **Implemented** to **Not Implemented**. The rater selects one score from among the following choices:

3--Implemented, meaning the teacher's performance aligns with the descriptor,
2+

2--Partially Implemented, meaning the teacher’s instruction reflects this item but there are flaws or missing components in the way in which it is implemented,

2-

1--Not Implemented, meaning the item is either implemented poorly or should have been observed but is not,

and **Not Applicable (N/A)**, a category that recognizes that given the lesson context and what is taught in previous lessons, not every item will be observed across every observation.

Preparation for the Observation

There are several materials you will need in order to use the WPS to conduct the observations. First, you should ensure you have everything you need to conduct the observation including pencils, a clipboard (or something hard to write on), and a copy of the WPS. The WPS is your scoring form and your note-taking space. Use the margins and the backs of pages to write notes of the things you observe that help you determine what ratings to assign. The notes will be also useful when you need to provide feedback to the teacher.

Understanding the WPS Structure

There are 17 items in this rubric. Each item is listed in a table below with an explanation and description of the intention of the item to help clarify its meaning. Each item has five levels of implementation. Descriptors are given for high, middle, and low levels of implementation. Examples are included to help you interpret the meaning of the different implementation levels. You should consider these descriptions and examples as you determine the implementation level for each item.

Assigning Rating on WPS

The WPS rating scale includes a score of: **1) “Not Implemented,” 2) “Partially Implemented,” and 3) “Implemented,”**. The “Partially Implemented” category is further divided to allow for assigning a **2-**, a **2**, or a **2+**, to indicate the degree to which the item is partially implemented. A **2-** indicates a very low level of partial implementation, whereas a **2+** can be used in cases where the item is almost fully implemented but not quite.

Observing either live or from video, you assign a rating on the basis of the observations. Assign a rating that comes closest to describing the observation even if not an exact match. For each item, assign a single rating, unless it is N/A.

Because the duration of a class may be 40 minutes or more, it is helpful to note whatever is observed, even at a low level. Then if a higher level item implementation makes the previous item inaccurate, the previous choice can simply be changed. This is especially useful when some items need to be observed throughout the whole lesson. For example, after observing the teacher “allows adequate time for students to think or respond,” the observer should check ‘Partially Implemented’, but if the teacher continues to allow adequate time for students to think or respond in that way until the end of the lesson, ‘Partially Implemented’ should be crossed out and a higher level of item implementation is checked.

Description of the WPS rubric

Component 1: Content/Design of Instruction

The section describes the critical elements of schema-based instruction that research has identified as effective in developing students’ abilities to solve word problems.

Elements of Component 1 are:

Item 1-The lesson consistently focuses on critical schema(ta) that will transfer/generalize to a range of examples.

Item 2-The teacher guides students to use schematic diagrams to assist with identifying the schema and salient information.

Item 3-The teacher presents a range of examples that supports generalization of the schema.

Item 4-The teacher guides students to use a mnemonic, heuristic, or procedural strategy checklist to support the problem solving process as needed.

Component 2: Sequence of Instruction

This component describes the sequence of instruction across lessons.

Elements of Component 2 are:

Item 5-The teacher presents one new schema in a lesson.

Item 6-The teacher introduces new problem schemas in story situations with no unknown quantities.

Item 7-Once students demonstrate understanding of schema, the teacher shifts instruction to solving processes (story situations now include an unknown quantity.)

Item 8-Once a new schema is mastered, the teacher contrasts it with previously learned schema(ta).

Item 9-The teacher introduces two-step word problems with previously mastered schemata.

Component 3: Delivery of Instruction

This component contains items that describe the manner in which the teacher delivers the instruction. This includes the ways in which the teacher is responsive to students' needs and the quality of the teacher's communication.

Elements of Component 3 are:

Item 10-The teacher clearly and sufficiently verbalizes and models reasoning for the schema and/or solution process.

Item 11-The teacher engages students in making connections between schema, diagrams, meanings of operations, and procedures.

Item 12-The teacher effectively reviews or teaches key vocabulary and/or symbols.

Item 13-The teacher consistently discusses mathematical ideas with language that is clear, accurate, and precise.

Component 4: Student Engagement

This component contains items that describe how the teacher has planned for and implements opportunities for students to be engaged in the lesson and practice working with the concept.

Elements of Component 4 are:

Item 14-The teacher provides students with sufficient opportunity to verbalize their understanding and/or reasoning.

Item 15-The teacher encourages students to use mathematical vocabulary and/or symbols throughout the lesson.

Item 16-The teacher provides students with practice adequate to supporting the development of understanding of the schema or problem solving process.

Component 5: Providing Feedback

This component contains items that describe the nature of the feedback provided to students.

Elements of Component 5 are:

Item 17-Feedback is consistently linked to mathematical reasoning and schema.

Psychometric Properties--*under development*

The WPS rubric has been developed through a rigorous process to ensure that it is a valid and reliable instrument. Each item included within the rubric comes from an analysis of the existing research establishing schema-based instruction as an effective practice for students with high incidence disabilities. Additionally, the rubric has been reviewed by content experts in the field to support content validity.

Further psychometric review is ongoing.

Key Terms on the WPS

“**Adequate**” means as much or as good or as necessary to accomplish a purpose or produce intended or expected results.

“**Consistently**” means every time the opportunity arises, the teacher responds in the same or an appropriately similar way. It is different from continuously.

“**Effectively**” means adequate to accomplish a purpose or produce intended or expected results.

“**Sufficiently**” means with enough frequency, depth, or explicitness to accomplish a purpose.

Item Descriptions and Examples

<p>Item 1: The lesson consistently focuses on critical schema(ta) that will transfer/generalize to a range of examples.</p> <p>This item addresses the need for students to approach problem solving with a knowledge of and using the mathematical relationships and concepts that drive problem solving processes. Critical schemata are those that help reflect the underlying mathematical structure and generalize across contexts and numbers. See pages 1-2 for more information about schema.</p>		
Implemented-3	Partially Implemented-2	Not Implemented-1
<p>The lesson consistently focuses on critical schema(ta) that will transfer/generalize to a range of examples.</p>	<p>The lesson inconsistently focuses on critical schema(ta) that will transfer/generalize to a range of examples.</p>	<p>The lesson does not focus on critical schema(ta) that will transfer/generalize to a range of examples.</p>
<p>Examples:</p> <ul style="list-style-type: none"> • The teacher presents a lesson during the problem solving phase focused on the schema of change problems (change-result unknown, change-change unknown, change-start unknown). • The teacher focuses a lesson on the equal groups schema. The students use a heuristic (CUBES) to guide their problem solving processes, but instruction remains focused on understanding the equal groups structure. 	<p>Examples:</p> <ul style="list-style-type: none"> • The teacher presents a lesson change problems, but the teacher switches from discussing the schema of change problems (with start, change, result) to talking about addition and subtraction. • The teacher presents a lesson on the equal groups schema. However, considerable time is put on asking students to use a heuristic (CUBES) to guide their problem solving. This limits the amount of time that the teacher can spend on equal groups. 	<p>Examples:</p> <ul style="list-style-type: none"> • The teacher presents a lesson on word problems with addition only focusing on key words like “altogether” and “how much more”. • Instruction is focused on applying a general problem solving heuristic (CUBES) to many different problem types.

Item 2-The teacher guides students to use schematic diagrams to assist with identifying the schema and salient information.

This item evaluates whether the teacher is scaffolding students' abilities to represent the word problems using schematic diagrams. The schematic diagrams are models of the underlying mathematical concepts and relationships within the word problem. The schematic diagrams are not representations of the specific contexts or individuals involved in a particular word problem; they are generalizable to the schema(ta) that is/are the focus of the lesson. Guidance involves a gradual release of responsibility for representing word problems from teacher to student. Guidance can take the form of explicit modeling, gradual release of steps in the process, prompts, questions, etc.

Implemented-3	Partially Implemented-2	Not Implemented-1
<p>The teacher guides students to use schematic diagrams to assist with identifying the schema and salient information.</p>	<p>The teacher provides some guidance to students for using schematic diagrams but more is needed.</p>	<p>The teacher does not guide students to use schematic diagrams.</p>
<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher provides an explicit model of the equal groups schematic diagram (unit rate, number of units, product) for two word problems. Then the teacher prompts students to apply the same schematic diagram to a third word problem. ● The teacher gradually releases responsibility for drawing the diagram. After modeling the first problem, the teacher draws the schematic diagram. Then the teacher asks students, "Which sentence tells about the combined quantity?" Then the teacher puts this quantity in the appropriate location. On the next example, the teacher asks a student to fill in the diagram. 	<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher provides students with an appropriate schematic representation of equal groups problems, but the teacher does not gradually release control to students for representing the word problems in these diagrams. 	<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher guides students to draw visual representations that directly model the specific contexts of the word problems (apples in baskets, cookies on trays) rather than the equal groups schema.

Item 3-The teacher presents a range of examples that supports generalization of the schema.

This item evaluates whether the examples presented by the teacher support students' abilities to identify the mathematical relationship across multiple contexts. The teacher should begin with simple number relations and familiar contexts, gradually selecting more difficult numbers and unfamiliar contexts. During the problem solving phase of instruction, the teacher should vary the placement of the unknown number. All examples should align with the schema that the teacher is focusing on.

Implemented-3	Partially Implemented-2	Not Implemented-1
<p>The teacher presents a range of examples that support generalization of the schema.</p>	<p>The teacher presents a range of examples that somewhat support generalization of the schema.</p>	<p>The teacher does not present a range of examples that support generalization of the schema, OR the teacher presents examples that are not aligned to the schema creating confusion.</p>
<p>Examples:</p> <ul style="list-style-type: none"> • The teacher begins with simple numbers in a familiar context. Next the teacher includes larger numbers in the same context. Once students demonstrate success, the teacher moves to less familiar contexts. • The teacher demonstrates an additive compare schematic diagram with examples with unknown quantities in different positions (larger, smaller, and difference). Teacher begins with the difference unknown before changing the unknown position. 	<p>Examples:</p> <ul style="list-style-type: none"> • The teacher begins with simple numbers in a familiar context. The teacher then moves to larger numbers and similar contexts. The teacher does not include different or unfamiliar contexts. • The teacher demonstrates an additive compare schematic diagram with several examples (larger, smaller, and difference). The teacher does not change the position of the unknown. 	<p>Examples:</p> <ul style="list-style-type: none"> • The teacher provides examples with a very limited range of numbers within the same context. Students are not exposed to variations of the schema. • The teacher demonstrates an additive compare schematic diagram. The teacher mistakenly includes examples from other change schema problems creating confusing.

Item 4-The teacher guides students to use a mnemonic, heuristic, or procedural strategy checklist to support the problem solving process as needed.

This item evaluates whether the teacher is scaffolding students' problem solving process with general guidelines that apply across all problems. Providing these supports is important for students who may have disabilities such as memory or language processing. The use of these techniques should not distract from or conflict with instruction centered on schema.

Guidance involves a gradual release of responsibility for representing word problems from teacher to student. Guidance can take the form of explicit modeling, gradual release of steps in the process, prompts, questions, etc.

Implemented-3	Partially Implemented-2	Not Implemented-1
<p>The teacher guides students to use a mnemonic, heuristic, or procedural strategy checklist to support the problem solving process as needed.</p>	<p>The teacher provides some guidance to use a mnemonic, heuristic, or procedural strategy checklist to support the problem solving process but more is needed.</p>	<p>The teacher does not guide students to use a mnemonic, heuristic, or procedural strategy checklist to support the problem solving process as needed, OR the support provided does not align with schema-based instruction.</p>
<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher provides students with a prompt card with questions that guide identification of salient numbers. Students are familiar with the questions and use the card as needed. ● Throughout the lesson, the teacher clearly models a simple mnemonic that guides students through the problem solving steps. 	<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher provides students with a prompt card with questions that guide identification of salient numbers. Some students forget to use the card, and the teacher does not consistently prompt them as needed. ● The teacher is modeling the use of a mnemonic for the problem solving process, but she does so inconsistently, sometimes forgetting steps. 	<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher provides students with a prompt card with questions that guide identification of salient numbers. Students are not familiar with the questions and do not know how to use the card. The teacher does not provide instruction. ● The teacher models a mnemonic that centers on finding key words in the word problem.

Item 5-The teacher presents one new schema in a lesson.

This item ensures that students are given the opportunity to develop a strong understanding of new schema before other schema are introduced.

This item may be scored N/A if instruction has moved on to the problem solving phase, contrasting previously learned schemata, or two-step problem solving.

Implemented-3	Partially Implemented-2	Not Implemented-1
The teacher presents one new schema in a lesson.		The teacher presents more than one new schema in the same lesson.
Examples: <ul style="list-style-type: none">• The teacher presents one new schema--group problems for addition and subtraction (part-part-whole).		Examples: <ul style="list-style-type: none">•The teacher presents two new schema for addition and subtraction--change problems and group problems(part-part-whole).

Item 6-The teacher introduces new problem schemas in story situations with no unknown quantities.

This item ensures that instruction provides students with the opportunity to focus on the schema and not be distracted by trying to find the answer. This also applies to instruction in two-step word problems.

This item may be scored N/A if instruction has moved on to the problem solving phase or when contrasting previously learned schemata.

Implemented-3	Partially Implemented-2	Not Implemented-1
The teacher introduces new problem schemas in story situations with no unknown quantities.		The teacher introduces new problem schema in story situations with unknown quantities.
<p>Examples:</p> <ul style="list-style-type: none"> • The teacher introduces equal groups schema with a problem with a missing quantity. (There are 4 bags, and there are three cookies in each bag. How many cookies are there?) Students focus on finding the answer of 12 rather than on the equal groups schema. 		<p>Examples:</p> <ul style="list-style-type: none"> • The teacher introduces equal groups schema with a problem with no missing quantities. (There are 4 bags, and there are three cookies in each bag. There are 12 cookies.) Students focus attention on the unit rate and the number of units.

Item 7-Once students demonstrate understanding of schema, the teacher shifts instruction to solving processes (story situations now include an unknown quantity.)

This item ensures that students understand the schema and can identify the components before turning instruction to focus on selecting the appropriate solution plan. This also applies to instruction in two-step word problems.

This item may be scored N/A if instruction is in the understanding schema phase or when contrasting previously learned schema.

Implemented-3	Partially Implemented-2	Not Implemented-1
<p>Once students demonstrate understanding of schema, the teacher shifts instruction to solving processes (story situations now include an unknown quantity.)</p>	<p>The teacher shifts instruction to solving processes before verifying student understanding of schema.</p>	<p>The teacher includes both understanding of the schema and instruction in solving processes at the same time.</p>
<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher verifies that students can correctly identify the components of the schema with a schematic diagram before beginning to model problems with unknowns and how to determine a solution plan. 	<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher provides several examples with no unknown quantities but does not verify that students can apply the schematic diagram to a new problem correctly before moving on to problem solving with unknowns. 	<p>Examples:</p> <ul style="list-style-type: none"> ● After introducing the schema with no unknown quantities, the teacher shifts to include problem solving processes. The teacher is modeling how to complete the schematic diagram and solve the problem at the same time.

Item 8-Once a new schema is mastered, the teacher contrasts it with previously learned schema(ta).

The purpose of this item is to ensure that students have the opportunity to contrast previously learned schemata. This helps students further develop understanding of the schemata, identify the correct schemata in new situations, and provides ongoing practice. This also applies to instruction on two-step word problems with previously practiced structures. This item may be scored N/A if instruction is still in the understanding schema or problem solving phases.

Implemented-3	Partially Implemented-2	Not Implemented-1
<p>Once a new schema is mastered, the teacher contrasts it with previously learned schema(ta).</p>		<p>The teacher does not contrast mastered schema with schema(ta) that were previously learned.</p>
<p>Examples:</p> <ul style="list-style-type: none"> • The teacher verifies students can correctly solve a problem with the groups schema from the previous lesson before this to (previously learned) change schema. The lesson focuses on correctly identifying these schema from a number of examples. 		<p>Examples:</p> <ul style="list-style-type: none"> • The teacher does not verify students can correctly solve a problem with the groups schema from the previous lesson before this to (previously learned) change schema. • The teacher does not contrast learned schema. Instead the teacher moves directly to new schema.

Item 9- The teacher introduces two-step word problems with previously mastered schemata.

This item focuses on using previously learned mathematical relationships to chain together multiple steps in a word problem. It ensures new schema are not added when working with multi-step problems.

This item may be scored N/A if instruction is in the understanding schema, problem solving, or contrasting learned schema phases.

Implemented-3	Partially Implemented-2	Not Implemented-1
The teacher introduces two-step word problems with previously mastered schemata.		The teacher introduces two-step word problems with schema that are not yet mastered by the students.
<p>Examples:</p> <ul style="list-style-type: none"> • The teacher uses previously learned group schema (part-part-whole) and equal group schema (unit rates) in multi-step problems. 		<p>Examples:</p> <ul style="list-style-type: none"> • The teacher includes two step problems when first teaching the equal group schema.

Item 10-The teacher clearly and sufficiently verbalizes and models reasoning for the schema and/or solution process.

This item focuses on how the teacher makes the concepts and the reasoning behind them explicit and clear for the students. This item does not include engaging in questioning of the students; this item is focused on the teacher clearly and sufficiently communicating thought processes through explanation and think-aloud. Explanation and think-aloud proceeds step-by-step to provide reasons for decision making and procedures. This is accomplished by using visual representations, schematic diagrams, modeling actions, and by carefully and thoroughly articulating the thinking processes involved in the example. Modeling may include gestures as appropriate (e.g., sweep of hand to different sides of an equation).

Implemented-3	Partially Implemented-2	Not Implemented-1
<p>The teacher clearly and sufficiently verbalizes and models reasoning for the schema and/or solution process.</p>	<p>The teacher verbalizes and models reasoning for the schema and/or solution process but not clearly and sufficiently.</p>	<p>The teacher does not verbalize and model reasoning for the schema and/or solution process.</p>
<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher presents an organized and thorough explanation of each step and think-alouds that reveal reasons for each decision made during the problem solving process. 	<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher provides explanations, models, and think-alouds but these are not always clear and consistent across problems. The teacher does not always explain how she knows a feature of the story problem is important. ● The teacher carefully explains the first example but proceeds too quickly through the second. Students appear to have trouble following along. 	<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher's presentation is focused on steps in the procedure without attempting to explain the basis or reasons for the steps.

Item 11-The teacher engages students in making connections between schema, diagrams, meanings of operations, and procedures.

This item ensures that connections are an explicit part of the lesson. To be most effective in developing understanding, students need to see the connections, describe them, use them, and apply them in new problems. Engaging students in making these connections also gives students additional opportunities to practice. Engaging students in making connections can take many forms: think-pair-share, practice that involves modeling and/or explaining, asking for verbal or written responses or for gestures or actions, etc.

Implemented-3	Partially Implemented-2	Not Implemented-1
<p>The teacher engages students in making connections between schema, diagrams, meanings of operations, and procedures.</p>	<p>The teacher demonstrates connections between schema, diagrams, meanings of operations, and procedures but does not engage students, OR the connections are limited due to missed opportunities.</p>	<p>The teacher does not make connections between schema, diagrams, meanings of operations, and procedures.</p>
<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher demonstrates the schematic diagram for multiplicative comparison problems. Then the teacher leads a discussion with purposeful questions making the similarities between two examples explicit. ● During a lesson contrasting two learned schemata, teacher demonstrates similarities and differences. Then the teacher asks students to discuss these with a partner and new examples. 	<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher demonstrates the schematic diagram for multiplicative comparison problems. The teacher explains the schema to the students without engaging students in the discussion. ● The teacher discusses the connections between the schematic diagram and the procedure but misses an obvious opportunity to connect to the meaning of subtraction. ● The teacher only presents two examples. This limits opportunities to describe connections and to involve students. 	<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher draws the schematic diagram on the board but does not make connections to the context or meaning of operations explicit. The teacher seems to assume these connections are obvious.

Item 12-The teacher effectively reviews or teaches key vocabulary and/or symbols.

It is important to ensure that the meanings of vocabulary and symbols are clear to students. Ensuring this clarity is important for students with disabilities such as memory or language processing rather than assuming they will remember or infer the meanings. This may occur at the beginning of a lesson or may occur mid-way through a lesson as appropriate. Students may provide the review if they are able to provide clear and accurate definitions and/or examples. If they cannot, the teacher should provide clear and explicit definitions. Effectively reviewing or teaching involves a clear, timely, concise, focused explanation of the term or symbol.

Implemented-3	Partially Implemented-2	Not Implemented-1
<p>The teacher effectively reviews or teaches key vocabulary and/or symbols.</p>	<p>The teacher reviews or teaches key vocabulary and/or symbols but not effectively, OR the teacher reviews or teaches some key vocabulary and/or symbols.</p>	<p>The teacher does not review or teach key vocabulary and/or symbols.</p>
<p>Examples:</p> <ul style="list-style-type: none"> ● During a lesson on the multiplicative comparison schema, the teacher asks a students to give the meaning “schematic diagram”. The student gives a concise definition. The the teacher demonstrates the diagram for the schema providing a clear, student-friendly definition of “referent unit”. ● The teacher defines all the terms in the schematic diagram. 	<p>Examples:</p> <ul style="list-style-type: none"> ● During a lesson on the equal groups schema, the teacher asks a students to give the meaning “schematic diagram”. The students are not able to give a clear definition and the teacher tries to provide hints or have students guess. ● The teacher provides a complicated and confusing definition of “referent unit.” ● The teacher reviews the term unit but does not review the term unit rate. 	<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher does not review or teach relevant key vocabulary including part, whole, group, and diagram. The teacher only asks students what the word “add” means and accepts an answer limited to “when you get more”.

Item 13-The teacher consistently discusses mathematical ideas with language that is clear, accurate, and precise.

This item focuses on the language used by the teacher during the instruction. The teacher uses unambiguous wording and academic terminology based on the students' receptive vocabulary. Students need to have concepts presented with language that is academic, consistent, and appropriate for the students if they are to use and apply those concepts.

Implemented-3	Partially Implemented-2	Not Implemented-1
<p>The teacher consistently discusses mathematical ideas with language that is clear, accurate, and precise.</p>	<p>The teacher discusses mathematical ideas with language that is clear, accurate, and precise but not consistently.</p>	<p>The teacher does not discuss mathematical ideas with language that is clear, accurate, and precise.</p>
<p>Examples:</p> <ul style="list-style-type: none"> ● In response to a student, the teacher says, “Yes, to solve this we are doing ‘timesing’. In math, the name for ‘timesing’ is ‘multiplying’.” The teacher continues to use the word “multiplying” and reminds student of the term when they use “timesing.” ● The teacher is consistent in using terms such as units, unit rate, product and connects them to the word problems clearly, accurately, and precisely. 	<p>Examples:</p> <ul style="list-style-type: none"> ● In response to a student, the teacher says, “Yes, to solve this we are doing ‘timesing’. That is the same as multiplying.” The teacher does not consistently use the term ‘multiplying’ later in the lesson. ● The teacher is consistent with some terms (units, our units are apples) but not with others (unit rate, per, each used interchangeably but not clearly connected). 	<p>Examples:</p> <ul style="list-style-type: none"> ● In response to a student, the teacher says, “Yes. We are ‘timesing’.” The teacher uses the word ‘timesing’ throughout the rest of the lesson. ● The teacher discusses the schema with inconsistent and imprecise language, switching between terms without clarifying meaning: units, unit rate, per, each, bags of apples, items, apples, etc..

Item 14-The teacher provides students with sufficient opportunity to verbalize their understanding and/or reasoning.

This item assesses whether students are given an opportunity to communicate their understanding and reasoning. This goes beyond simply providing an answer to a math problem. While teachers may scaffold this by modeling explanations and analyzing their own and others thinking, this item looks specifically at the opportunity for students to communicate their own thinking. Opportunities to verbalize may include asking students to think-aloud, summarize, answer questions, agree/disagree, explain or elaborate.

Implemented-3	Partially Implemented-2	Not Implemented-1
<p>The teacher provides students with sufficient opportunity to verbalize their understanding and/or reasoning.</p>	<p>The teacher provides students with limited opportunity to verbalize their understanding and/or reasoning.</p>	<p>The teacher does not ask students to verbalize their understanding and/or reasoning.</p>
<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher frequently asks students to explain their reasoning rather than just provide an answer. For example, she says, “Tell me your thought process here,” and when a student gives a one-word answer, she presses by asking, “Why?” ● The teacher asks questions and checks for understanding. The teacher then provides another example and asks students to explain their thinking process to one another after solving. The teacher prompts students to explain their thinking. 	<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher asks students to tell her what they did to solve a problem but does not ask them to explain their reasoning. ● There is some questioning observed during the lesson that enables students to respond orally. However, the teacher provides most of the discussion and examples. ● The teacher asks questions and checks for understanding of the students who offer an answer but not for other students. 	<p>Examples:</p> <ul style="list-style-type: none"> ●●The teacher asks for thumbs up/thumbs down. This is done throughout the lesson, but this is the only means students have for communicating their understanding. ● The teacher asks students for answers, but does not ever ask, “How do you know?” ● The teacher calls on students, but feeds them the answers to a degree that it isn’t clear how much students are able to answer on their own.

Item 15-The teacher encourages students to use mathematical vocabulary and/or symbols throughout the lesson.

This item assesses whether the teacher provides opportunities for students to be actively engaged with the terminology and symbols that are important to the subject of mathematics. Opportunities to respond should occur frequently throughout the lesson.

Implemented-3	Partially Implemented-2	Not Implemented-1
<p>The teacher encourages students to use mathematical vocabulary and/or symbols throughout the lesson.</p>	<p>The teacher encourages students to use mathematical vocabulary and/or symbols but not consistently throughout the lesson.</p>	<p>The teacher does not encourage students to use mathematical vocabulary and/or symbols.</p>
<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher defines the term “unit rate” and asks students to repeat the term. Throughout the lesson, the teacher encourages students to use the term “unit rate” accurately and appropriately. 	<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher defines a term “unit rate” and asks students to repeat the term. Though the teacher compliments students who use the term throughout the lesson, she does not consistently ask students to use the term when appropriate. 	<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher defines a term “unit rate” but never asks students to use it.

Item 16-The teacher provides students with practice adequate to supporting the development of understanding of the schema or problem solving process.

This item assesses whether students have the opportunity to practice first using the schematic diagram and then going through the problem solving process with unknown quantities. To support understanding, students should practice with a range of examples that is appropriate for their skill level. A range of examples can include may include number sets with increasing difficulty, as appropriate. The examples should also include familiar and unfamiliar contexts.

Implemented-3	Partially Implemented-2	Not Implemented-1
<p>The teacher provides students with practice adequate to supporting the development of understanding of the schema or problem solving process.</p>	<p>The teacher provides students with practice somewhat adequate to supporting the development of understanding of the schema or problem solving process.</p>	<p>The teacher provides students with practice inadequate to supporting the development of understanding of the schema or problem solving process.</p>
<p>Examples:</p> <ul style="list-style-type: none"> ● After a demonstration of using the schematic diagram with no unknown quantities, students practice with a worksheet with several problems that provides space and prompts for constructing schematic diagrams with additional examples. ● Students begin guided practice but are struggling due to complexity of the task. The teacher adjusts and provides simpler practice that allows students to develop understanding of the schema and be successful with the schematic diagram. 	<p>Examples:</p> <ul style="list-style-type: none"> ● After a demonstration of using the schematic diagram with no unknown quantities, students practice with a worksheet that only includes two opportunities to apply the diagram. Despite time for more, the teacher directs students who finish early to alternative activities. ● Students begin guided practice but are struggling due to complexity of the task. After growing confusion, the teacher adjusts and provides scaffolds that support their ability to complete the task as given. The scaffolding only supports 	<p>Examples:</p> <ul style="list-style-type: none"> ● Students are only given an opportunity to practice with one example during the class. ● The teacher gives a worksheet that includes number sets more complicated than those presented earlier in the lesson. ● The teacher gives a worksheet with number sets that are too simple. Students finish quickly and do not have an opportunity to transfer the skill to new situations.

	<p>getting the right answer, and therefore practice that enhances understanding of the schema is minimal.</p>	<ul style="list-style-type: none">● Rather than providing scaffolds when students are having difficulty, the teacher tells the students what numbers go in the blanks on their worksheets.
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Item 17-Feedback is consistently linked to mathematical reasoning and schema.

This item evaluates the focus of feedback. Feedback may include specific information about reasoning, diagrams, processes, or calculations. Feedback can take the form of correction, suggestion, prompting, cueing or reinforcing and affirming. Both the nature and complexity of the task and the lesson goal need to be taken into consideration.

Implemented-3	Partially Implemented-2	Not Implemented-1
<p>Feedback is consistently linked to mathematical reasoning and schema.</p>	<p>Feedback is not consistently linked to mathematical reasoning and schema.</p>	<p>There is no feedback, OR feedback is not linked to mathematical reasoning and schema.</p>
<p>Examples:</p> <ul style="list-style-type: none"> ● When the student places 6 and 2 in the wrong places in the diagram, the teacher gives corrective feedback and describes it as groups (units) and size of group (unit rate) reflecting the language used in the lesson. ● Students are successful with the tasks. The teacher takes several opportunities to give informative feedback such as, “Good. You saw that that 5 was the size of your group or unit rate, and that you had 4 groups--the number of units.” 	<p>Examples:</p> <ul style="list-style-type: none"> ● Feedback is specific and informative when students make a mistake, but affirmative feedback is not specific. For example, a student writes $6 + 2$ instead of 6×2. The teacher gives corrective feedback. When the student is successful with the next problem of 4×3, the teacher just says, “You got it.” The teacher could say, “Good. You knew that you were modeling groups or units and size of group or unit rate.” 	<p>Examples:</p> <ul style="list-style-type: none"> ● The teacher consistently only tells students if they are right or wrong. If they are wrong, the teacher says, “Read it again.” ● The teacher simply states that students are correct or prompts students to perform steps in the procedure. The teacher does not discuss the meaning of the steps. ● Students are successful with the tasks. The teacher only tells students, “Good job.”

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