# Practicing Computation Rubric Manual 

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## Practicing Computation Mathematics Rubric

## Overview:

An important goal for students is to develop computational fluency which is defined by the National Council of Teachers of Mathematics (NCTM, 2000) as "having efficient and accurate methods for computing" (p. 152). Frequent practice helps students develop computational fluency. Practicing computation refers to applying knowledge of strategies and procedures to mathematical problems. Practice helps students maintain recently acquired knowledge, retain previously learned material, build fluency or automaticity, and later supports connections with new and more complex content (Doabler et al., 2015; Pellegrino \& Goldman, 1987; Prawat, 1989). Along with the goals of developing efficiency and accuracy, a further goal is to develop independence.

To achieve these goals, the mathematical strategies employed by students may vary depending on the students' cognitive characteristics. Often, strategy use is more efficient than rote memorization because strategies may also help students develop flexibility and the ability to see connections, and strategies may apply across many novel situations. Strategy use increases the likelihood that students will apply what they've learned in new situations, both in and out of school (Rittle-Johnson \& Alibali, 1999).

The Practicing Computation rubric applies to practice of both newly acquired skills and strategies and review of previously learned skills and strategies. Students with difficulty in mathematics or with learning disabilities in mathematics benefit from frequent well-designed review of newly acquired and previously learned skills and strategies that is spaced over time (Doabler et al., 2018).

The Practicing Computation rubric was designed for use by supervisors and administrators to reliably evaluate teachers' implementation of practices that support students' development of computational fluency. The rubric provides specific, accurate, and actionable feedback to teachers about the quality of their instruction and, ultimately, improve the outcomes for students with or at-risk for mathematics difficulty. The purpose of this manual is to provide technical information for implementing the Practicing Computation rubric as a tool for evaluation and feedback.

This rubric includes 4 components. These are:

1) Design of Lesson
2) Delivery of Lesson
3) Student Engagement
4) Providing Feedback

Under these 4 components, there are 10 items. For most items, there are five levels of implementation. One item has two levels--implemented or not implemented. Observing either live or from video, the observer assigns a rating based on a scale that ranges from Implemented to Not Implemented. The rater selects one score from among the following choices:
3--Implemented, meaning the teacher's performance fully aligns with the descriptor, 2+
2--Partially Implemented, meaning the teacher's instruction reflects this item but there are flaws or missing components in the way in which it is implemented, 2-
1--Not Implemented, meaning the item is either implemented poorly or should have been observed but is not,
and Not Applicable (N/A), a category that recognizes that given the lesson context and what is taught in previous lessons not every item will be observed across every observation.

## Preparation for the Observation

There are several materials you will need in order to use the Practicing Computation rubric to conduct the observations. First, you should ensure you have everything you need to conduct the observation including pencils, a clipboard (or something hard to write on), and a copy of the Practicing Computation rubric. The Practicing Computation rubric is your scoring form and your note-taking space. Use the margins and the backs of pages to write notes of the things you observe that help you determine what ratings to assign. The notes will be also useful when you need to provide feedback to the teacher.

## Understanding the Rubric Structure

There are 10 items in this rubric. Each item is listed in a table in this manual with an explanation and description of the intention of the item to help clarify its meaning. Most
items have five levels of implementation (one item has only two levels--implemented or not implemented). Descriptors are given for high, middle, and low levels of implementation. Examples are included to help you interpret the meaning of the different implementation levels. You should consider these descriptions and examples as you determine the implementation level for each item.

## Assigning Ratings

The Practicing Computation rubric rating scale includes a score of: 1)"Not Implemented," 2) "Partially Implemented," and 3) "Implemented,". The "Partially Implemented" category is further divided to allow for assigning a 2-, a 2, or a $2+$, to indicate the degree to which the item is partially implemented. A 2-indicates a very low level of partial implementation, whereas a $2+$ can be used in cases where the item is almost fully implemented but not quite.

Observing either live or from video, you assign a rating on the basis of the observations. Assign a rating that comes closest to describing the observation even if not an exact match. For each item, assign a single rating, unless it is N/A.

Because the duration of a class may be 40 minutes or more, it is helpful to note whatever is observed, even at a low level. Then if a higher level item implementation makes the previous item inaccurate, the previous choice can simply be changed. This is especially useful when some items need to be observed throughout the whole lesson. For example, after observing that the teacher "provides students with the opportunity to verbalize their understanding and/or reasoning," the observer should check 'Partially Implemented'. If the teacher continues to provide students with the opportunity to verbalize their reasoning and/or understanding as appropriate throughout the lesson, 'Partially Implemented' should be crossed out and a higher level of item implementation is checked.

## Description of the Practicing Computation rubric

## Component 1: Design of Lesson

The purpose of this section is to focus on the lesson content that research has identified as effective in developing computational fluency. It is important that students have practice with both new and previously learned skills.

## Element of Component 1:

Item 1 - The teacher provides students with practice of a target skill and cumulative review adequate for supporting fluent computation.

## Component 2: Delivery of Lesson

This component contains items that describe the way a teacher systematically presents the material. This includes the ways in which the teacher is responsive to students' needs and the quality of the teacher's communication.

## Elements of Component 2:

Item 2 - The teacher's presentation of examples and/or practice problems is systematic, increasing in complexity in response to the needs of the students.

Item 3 - When needed, the teacher provides clear and concise demonstrations of the procedures or strategies.

Item 4 - The teacher consistently discusses mathematical ideas with language that is clear, accurate, and precise.

## Component 3: Student Engagement

This component contains items that describe how the teacher has planned for and implements opportunities for students to be engaged in the lesson and practice working with the skills and/or strategies.

## Elements of Component 3 are:

Item 5 - Students use strategies, as needed, that are based on mathematical concepts and properties.

Item 6 - The teacher guides students, as needed, to use the most efficient strategy they can for a given problem.

Item 7 - The teacher provides students with sufficient opportunity to verbalize their understanding and/or explain their reasoning.

Item 8 - The teacher encourages students to use mathematical vocabulary and/or symbols throughout the lesson.

## Component 4: Providing Feedback

This component contains items that describe the nature of the feedback provided to students.

## Elements of Component 4 are:

Item 9 - Feedback is immediate (coming promptly from the teacher or by means for self-checking).

Item 10 - Feedback is frequently focused on mathematical concepts or strategies.

## Psychometric Properties

The Practicing Computation rubric has been developed through a rigorous process to ensure that it is a valid and reliable instrument. Each item included within the rubric comes from an analysis of the existing research establishing these instructional practices as effective for students with or at-risk for mathematics difficulty. Additionally, the rubric has been reviewed by content experts in the field to support content validity.

Further psychometric review is ongoing.

## Key Terms on the Practicing Computation Rubric

"Adequate" means as much or as good or as necessary to accomplish a purpose or produce intended or expected results.
"Consistently" means every time an appropriate opportunity arises, the teacher responds in the same or an appropriately similar way. It is different from continuously. "Frequently" means regularly or often.
"Sufficient" means enough in quantity and quality to accomplish a purpose.
"Visual representations" refer to concrete and pictorial representations of numbers, concepts, and relationships. Pictorial representations can be informal or formal. For the
purposes of this rubric, visual representations include, but are not limited to, concrete manipulatives, concrete or pictorial bar models, tape diagrams, pictures, icons, number lines, graphs, etc.

## Item Descriptions and Examples

## Item 1 - The teacher provides students with practice of a target skill and cumulative review adequate for supporting fluent computation.

This item assesses whether students have the opportunity to practice the procedure that has been the focus of recent lessons and also practice of previously learned skills. The goal is for students to develop accuracy and efficiency. Adequate practice would include several opportunities for success and incorporate a range of examples that is appropriate for students' skill levels. A range of examples may include number sets with increasing difficulty or different contexts, as appropriate. Students need multiple opportunities to practice previously learned skills as well. This practice helps them to retain these skills, discriminate across problem types, and connect new skills to those that were previously learned.

| Implemented-3 | Partially Implemented-2 | Not Implemented-1 |
| :---: | :---: | :---: |
| The teacher provides students with practice of a target skill and cumulative review adequate for supporting fluent computation. | The teacher provides students with practice of a target skill and/or cumulative review somewhat adequate for supporting fluent computation. | The teacher provides students with practice of a target skill and/or cumulative review inadequate for supporting fluent computation. |
| Examples: <br> - After a quick review of the procedure for multiplying 2-digit numbers, students practice with several multiplication problems. A few students need prompting, so additional practice is provided. After completing several problems correctly, the students are given review--practice adding and subtracting 2-digit numbers. <br> - The lesson begins with a review of math facts and a word problem. After this, students begin practice of multi-digit subtraction but are | Examples: <br> - After a quick review of the procedure for multiplying 2-digit numbers, students practice with two more multiplication problems. Some answer correctly, but a few students need prompting. Additional practice is not provided. Then students are given two review problems involving addition and two involving subtraction of 2-digit numbers. <br> - The lesson begins with a review of math facts and a word problem. After this students begin practice of | Examples: <br> - After a quick review of the procedure for multiplying 2-digit numbers, students practice with two more multiplication problems. Several students get the wrong answers. Additional practice is not provided. Then the lesson moves on to another topic with which students have had limited exposure. The lesson does not include successful practice. <br> -The lesson does not include any cumulative review. |


| struggling due to complexity <br> of the numbers. The teacher <br> adjusts, demonstrates, and <br> provides practice with simpler <br> number sets with which <br> students are successful. The <br> teacher gradually increases <br> the complexity of the <br> practice. | multi-digit subtraction but are <br> struggling due to complexity of <br> the numbers. The teacher <br> does not adjust the task and <br> has to visit each student to <br> provide support to complete <br> the problems. Some students <br> do not have enough <br> opportunity to practice <br> because they are waiting for <br> the teacher's help. | - The teacher gives a <br> worksheet with number sets <br> that are too simple. Students <br> finish quickly and do not <br> have an opportunity to <br> transfer the skill to new <br> contexts or more challenging <br> number sets. |
| :--- | :--- | :--- |
| - Students practice a few <br> unmastered addition facts <br> embedded in several <br> previously learned facts. | - Students practice <br> unmastered addition facts with <br> only a few previously learned <br> facts. There are not enough <br> previously learned facts to <br> ensure adequate practice. | Sumber of unmastice a <br> numed <br> addition facts. The number is <br> too many for students to <br> have adequate practice for <br> developing fluency with a <br> strategy or automaticity. |

Item 2 - The teacher's presentation of examples and/or practice problems is systematic, increasing in complexity in response to the needs of the students. This item examines whether the teacher provides deliberate, careful, and sequenced examples to support students' ability to calculate or apply a procedure correctly and efficiently. There is a clear and deliberate progression to the instruction that is responsive to the needs of students. The teacher increases the complexity after ensuring that students are successful at the current level of complexity.

| Implemented-3 | Partially Implemented-2 | Not Implemented-1 |
| :---: | :---: | :---: |
| The teacher's presentation of examples and/or practice problems is systematic, increasing in complexity in response to the needs of the students. | The teacher's presentation of examples and/or practice problems is somewhat systematic <br> OR <br> somewhat responsive to the needs of the students. | The teacher's presentation of examples and/or practice problems is not systematic OR <br> not responsive to the needs of the students. |
| Examples: <br> - The practice of fraction skills is ordered so that foundational skills are practiced first. The complexity of the denominator is gradually increased at each phase. Students identify fractions on a number line. Then students find equivalent fractions. Students practice addition with simple unit fractions ( $1 / 4$, $1 / 5,1 / 8$ ) After checking for accuracy, the teacher provides practice with one unit fraction and one non-unit fraction. <br> - The teacher has a set of practice problems of addition with regrouping that increase in complexity (regrouping in ones place with known facts, regrouping in ones place with | Examples: <br> - The practice of fraction skills includes related skills but not all are presented systematically. Students find equivalent fractions first. Then students practice with several problems with simple unit fractions. Then teacher then adds too much complexity at once with non-unit fractions and large denominators. <br> - The teacher has a set of practice problems that increase in complexity. The teacher gives students adequate practice with regrouping with known facts before moving to harder facts. However, the teacher moves to regrouping in the tens place when a few students needed support with the last example.The teacher could | Examples: <br> - The practice of fraction skills is not presented systematically. Topics are practiced and number sets selected in seemingly random order with more complex skills practiced before foundational skills. <br> - The teacher has a set of practice problems that increase in complexity. The students struggle with the first example, but the teacher moves on through each of the progressively more complex problems anyway. <br> - While students are practicing addition facts with flash cards, the teacher has them trade stacks with a partner every two minutes. |


| harder facts, regrouping in <br> tens place). The teacher <br> adds additional problems of <br> the same complexity as <br> needed before moving to the <br> next level. | add an additional example <br> before moving to greater <br> complexity or differentiate. | Practice is not systematic <br> and is not progressing in <br> response to needs of the <br> students. |
| :--- | :--- | :--- |
| - While students are |  |  |
| practicing addition facts, the |  |  |
| teacher adds new facts |  |  |
| gradually when other facts |  |  |
| are mastered maintaining a |  |  |
| balance of new and known |  |  |
| facts. |  |  |

## Item 3 - When needed, the teacher provides clear and concise demonstrations of the

 procedures or strategies.Students with difficulty in mathematics often need frequent demonstrations of procedures or strategies. However, because the ultimate goal is for students to be able to solve problems independently, the teacher should gradually withdraw supports as students demonstrate success. Therefore, there may be some components of the lesson, particularly parts of cumulative review, for which the teacher does not begin with a demonstration. The need for demonstrations may depend upon the topic, recency of previous practice, or individual student characteristics. Therefore, the teacher should be prepared to give a quick and clear demonstration as needed.
$\left.\begin{array}{|l|l|l|}\hline \text { Implemented-3 } & \text { Partially Implemented-2 } & \text { Not Implemented-1 } \\ \hline \begin{array}{l}\text { When needed, the teacher } \\ \text { provides clear and concise } \\ \text { demonstrations of the } \\ \text { procedures or strategies. }\end{array} & \begin{array}{l}\text { When needed, the teacher } \\ \text { provides demonstrations of the } \\ \text { procedures or strategies but not } \\ \text { clearly or concisely. }\end{array} & \begin{array}{l}\text { When needed, the teacher } \\ \text { does not provide } \\ \text { demonstrations of the } \\ \text { procedures or strategies. }\end{array} \\ \hline \begin{array}{l}\text { - The teacher reviews the } \\ \text { multiplication algorithm with a } \\ \text { clear presentation of a } \\ \text { step-by-step sequence of } \\ \text { actions. }\end{array} & \begin{array}{l}\text { - The teacher reviews the } \\ \text { multiplication algorithm but does } \\ \text { not communicate a clear } \\ \text { step-by-step procedure. }\end{array} & \begin{array}{l}\text { - The teacher does not } \\ \text { demonstrate when needed, } \\ \text { allowing students to } \\ \text { practice several problems } \\ \text { multiplication problems } \\ \text { incorrectly. }\end{array} \\ \text { - Seeing quickly that some } \\ \text { students do not remember } \\ \text { how to round numbers, the } \\ \text { teacher explains in an } \\ \text { organized, student-friendly } \\ \text { way with clear examples on a } \\ \text { number line. }\end{array} \quad \begin{array}{l}\text { - The teacher sees quickly that } \\ \text { some students do not } \\ \text { remember how to round } \\ \text { numbers. The teacher appears } \\ \text { to be unprepared and provides } \\ \text { an overly complicated verbal } \\ \text { explanation for rounding } \\ \text { numbers. }\end{array} \quad \begin{array}{l}\text { - The teacher reviews } \\ \text { some topics and not others } \\ \text { as needed. For example, } \\ \text { seeing that some students } \\ \text { do not remember how to } \\ \text { round numbers, the }\end{array}\right\}$

## Item 4-The teacher consistently discusses mathematical ideas with language that is clear, accurate, and precise.

This item focuses on the language used by the teacher during the instruction. The teacher uses unambiguous wording and academic terminology based on the students' receptive vocabulary. Students need to have mathematical concepts, procedures, and strategies, presented with language that is academic, consistent, and appropriate for the students if they are to use and apply mathematics.

| Implemented-3 | Partially Implemented-2 | Not Implemented-1 |
| :---: | :---: | :---: |
| The teacher consistently discusses mathematical ideas with language that is clear, accurate, and precise. | The teacher discusses mathematical ideas with language that is clear, accurate, and precise but not consistently. | The teacher does not discuss mathematical ideas with language that is clear, accurate, and precise. |
| Examples: <br> - The teacher uses the phrase "three-fourths" rather than "three out of four" (such as on a linear representation as compared to discrete items). The teacher is consistent in using terms such as parts, unit size, and whole. <br> - In response to a student, the teacher says, "Yes, we are doing 'timesing'. In math, the name for 'timesing' is 'multiplying'." The teacher continues to use the word "multiplying" and reminds student of the term when they use "timesing." <br> - When discussing angles, the teacher says, "Any figure may have more than one angle. The arc near the vertex is a symbol that tells you which angle to look at. Larger angles have longer arcs because the curve travels across an angle that is 'open wider'" | Examples: <br> - The teacher is consistent with some fraction terms (parts, unit size, whole) but not with others. The teacher says "three out of four" when the context is "three-fourths" (such as on a linear representation). <br> - In response to a student, the teacher says, "Yes, we are doing 'timesing'. That is the same as multiplying." The teacher does not consistently use the term 'multiplying' later in the lesson. <br> - When discussing angles, the teacher says, "You see the arcs in there. The larger the arc, the bigger the angle." (imprecise) | Examples: <br> - The teacher discusses fractions with inconsistent and imprecise language, switching between terms without clarifying meaning: parts, pieces, whole, one, size of whole, size of parts, etc. <br> - In response to a student, the teacher says, "Yes. We are 'timesing'." The teacher uses the word 'timesing' throughout the rest of the lesson. <br> - When discussing angles, the teacher says, "You see little circles in there. The larger the circle, the bigger the angle." (inaccurate and imprecise) |

## Item 5 - Students use strategies, as needed, that are based on mathematical concepts

 and properties.Strategies based on mathematical concepts and properties, rather than tricks, are applicable across problem types and number sets. Also, the use of strategies based on mathematical concepts and properties provides practice with mathematical reasoning and facilitates connections in the future. Mnemonics can be used appropriately and effectively (e.g., to support a sequence of steps) but should be supported by conceptual and/or procedural understanding.

| Implemented-3 | Partially Implemented-2 | Not Implemented-1 |
| :--- | :--- | :--- |
| Students use strategies, as <br> needed, that are based on <br> mathematical concepts and <br> properties. |  | Students do not use <br> strategies, as needed, that <br> are based on mathematical <br> concepts and properties. |
| Examples: <br> - As needed, students <br> practicing multiplication facts <br> use doubling to solve <br> multiplication by 4 and use a <br> decomposing strategy to solve <br> multiplication by 6. |  | Examples: <br> $\bullet$ The students use flash <br> cards and a few rhymes or <br> tricks to learn multiplication <br> facts. There is no practice <br> of strategies based on <br> mathematical reasoning. |

## Item 6-The teacher guides students, as needed, to use the most efficient strategy they can for a given problem.

This item examines questions and prompts used by the teacher as guidance. This guidance should support students' ability to apply strategies independently on future tasks. Strategies might include counting-up, decomposing, or recall for number facts, or using visual representations, heuristics, or mnemonics for other procedures. Guidance is an interaction between the teacher and students. The teacher provides students with adequate support as they apply an appropriate strategy to a given problem. The types of questions or prompts needed may vary depending upon the complexity and nature of the question or task and the students' current level of understanding; therefore, questions and prompts should be adjusted accordingly. Also, the teacher can guide students toward independent use of more efficient strategies, as appropriate for the student.

| Implemented-3 | Partially Implemented-2 | Not Implemented-1 |
| :---: | :---: | :---: |
| The teacher guides students, as needed, to use the most efficient strategy they can for a given problem. | The teacher guides students to use the most efficient strategy they can for a given problem, but more guidance is needed. | The teacher does not guide students to use the most efficient strategy they can for a given problem. |
| - The teacher reminds students they can use a decomposing strategy for a multiplication fact when students begin counting on their fingers. The teacher monitors to be sure they are using the strategy correctly. <br> - The teacher frequently asks students for the components of a mnemonic device when performing a multi-step procedure. | - When students encounter multiplication facts they have not memorized, the teacher asks, "What's a strategy you know?" The teacher does not follow up to be sure the strategy is used correctly. <br> - The teacher provides a mnemonic device for remembering the steps of a multi-step procedure but does not consistently prompt students to use it to monitor their own progress through a problem. Several times the teacher tells students which step they forgot. | - The teacher does not encourage students to use any more efficient strategies than counting on their fingers. <br> - The teacher does not provide any supports for remembering the steps of a multi-step procedure. <br> - The teacher does not question or prompt students to use a mnemonic for solving multi-digit multiplication problems. <br> - Rather than providing scaffolds when students are having difficulty, the teacher tells the students what numbers go in the |


|  |  | blanks on their <br> worksheets. |
| :--- | :--- | :--- |

## Item 7 - The teacher provides students with sufficient opportunity to verbalize their understanding and/or explain their reasoning.

This item assesses whether students are given an opportunity to communicate their understanding and reasoning. Opportunities to verbalize may include asking students to think-aloud, summarize, answer questions, agree/disagree, explain or elaborate. The goal is to give the teacher an opportunity to check for understanding and to reinforce correct reasoning. Sufficiency may be determined by whether all students are given the opportunity or whether the teacher is able to determine that students have clear and accurate understanding or reasoning. Opportunities to verbalize need not be so frequent that time is taken from opportunities to practice.
$\left.\begin{array}{|l|l|l|}\hline \text { Implemented-3 } & \text { Partially Implemented-2 } & \text { Not Implemented-1 } \\ \hline \begin{array}{l}\text { The teacher provides students } \\ \text { with sufficient opportunity to } \\ \text { verbalize their understanding } \\ \text { and/or explain their reasoning. }\end{array} & \begin{array}{l}\text { The teacher provides students } \\ \text { with limited opportunity to } \\ \text { verbalize their understanding } \\ \text { and/or explain their reasoning. }\end{array} & \begin{array}{l}\text { The teacher does not ask } \\ \text { students to verbalize their } \\ \text { understanding and/or } \\ \text { explain their reasoning. }\end{array} \\ \hline \begin{array}{l}\text { Examples: } \\ \text { - The teacher asks students } \\ \text { to explain their reasoning } \\ \text { rather than just provide an } \\ \text { answer. For example, she } \\ \text { says, "Tell me how you know," } \\ \text { or when a student gives a } \\ \text { one-word answer, she } \\ \text { presses by asking, "Why?" }\end{array} & \begin{array}{l}\text { Examples: } \\ \text { - The teacher asks students to } \\ \text { tell her what they did to solve a } \\ \text { problem but does not ask them } \\ \text { to explain their reasoning. }\end{array} & \begin{array}{l}\text { Examples: } \\ \text { - The teacher asks } \\ \text { students for answers to } \\ \text { math problems, but does } \\ \text { not ever ask, "How did } \\ \text { - The teacher asks questions } \\ \text { and checks for understanding of } \\ \text { the students who offer an } \\ \text { answer but not for other } \\ \text { students. }\end{array} \\ \begin{array}{ll}\text { "How do you know?" or }\end{array} \\ \text { - The teacher calls on } \\ \text { students, but feeds them } \\ \text { the answers to a degree } \\ \text { that it isn't clear how } \\ \text { much students are able to } \\ \text { answer on their own. }\end{array}\right\}$

## Item 8 - The teacher encourages students to use mathematical vocabulary and/or

 symbols throughout the lesson.This item assesses whether the teacher provides opportunities for students to be actively engaged with the terminology and symbols that are important to the subject of mathematics. Opportunities to respond should occur frequently throughout the lesson and may be aloud or in writing.

| Implemented-3 | Partially Implemented-2 | Not Implemented-1 |
| :--- | :--- | :--- |
| The teacher encourages <br> students to use mathematical <br> vocabulary and/or symbols <br> throughout the lesson. | The teacher encourages <br> students to use mathematical <br> vocabulary and/or symbols but <br> not consistently throughout <br> the lesson. | The teacher does not <br> encourage students to use <br> mathematical vocabulary <br> and/or symbols. |
| Examples: <br> - The students are practicing <br> multiplication within the <br> context of finding the area of <br> rectangles. Throughout the <br> lesson, the teacher <br> encourages students to use <br> the term "area" and provide <br> the units of measure <br> accurately and appropriately. | Examples: <br> - The students are practicing <br> multiplication within the context <br> of finding the area of <br> rectangles. Though the teacher <br> compliments students who use <br> the term throughout the lesson, <br> she does not consistently ask <br> students to use the term or <br> provide units of measure when <br> appropriate. | Examples: <br> - The students are <br> practicing multiplication <br> within the context of finding <br> the area of rectangles. The <br> teacher does not <br> encourage students to use <br> the term area or provide <br> the units of measure. |
| - The teacher consistently |  |  |
| prompts students to answer |  |  |
| "10 ones" instead of just |  |  |
| saying "ten." |  |  |$\quad$| - The teacher sometimes |
| :--- |
| accepts an answer of "10" |
| instead of "10 ones." |$\quad$| answer of "10" instead of |
| :--- |
| "10 ones" regularly. |

## Item 9 - Feedback is immediate (coming promptly from the teacher or by means for self-checking).

This item examines the teacher's timing of feedback as an element of the learning process. Affirmative or corrective feedback that is immediate can reinforce accuracy, prevent misconceptions or ensure accurate practice. Somewhat delayed feedback that allows the students time to think through the steps of a complex process may also be appropriate and support confidence, independence and self-regulation. When practicing or reviewing previously learned skills, feedback should be prompt enough to prevent students from practicing the skill incorrectly.

| Implemented-3 | Partially Implemented-2 | Not Implemented-1 |
| :---: | :---: | :---: |
| Feedback is immediate (coming promptly from the teacher or by means for self-checking). | Feedback sometimes inappropriately delayed. | Feedback is inappropriately delayed. |
| Examples: <br> - When practicing a recently learned skill, the teacher provides both affirmative and corrective feedback when the students complete the first problem. Once a student has completed a problem type successfully, the teacher checks in frequently to ensure they continue to be successful. When students have made a mistake, the teacher checks in as soon as they have completed the next problem. <br> -With a cumulative review worksheet, the teacher provides students with time to think and remember how to solve the problems. The teacher quickly reinforces | Examples: <br> - When practicing a recently learned skill, the teacher provides both affirmative and corrective feedback when the students complete the first problem. However, students who have made a mistake are allowed to complete the next several problems before the teacher checks their work. <br> -With a cumulative review worksheet, the teacher provides students with time to think and remember how to solve the problems. The teacher quickly corrects errors for some students but does not check all students' work promptly. <br> - While practicing math facts with cards, students have a | Examples: <br> - When practicing a recently learned skill, the teacher has students complete a set of three practice problems before checking their work. <br> -With a cumulative review worksheet, the teacher provides students with time to think and remember how to solve the problems. The teacher teacher does not provide feedback until they have finished the worksheet. <br> - While practicing math facts with cards, students complete a full set without any means for checking their answers. |


| correct thinking or corrects <br> errors. | procedure for self-checking <br> and returning missed facts to <br> the stack to practice again. The |  |
| :--- | :--- | :--- |
| - While practicing math facts |  |  |
| teacher inconsistently monitors |  |  |
| with cards, students have a |  |  |
| procedure for self-checking |  |  |
| and returning missed facts to |  |  |
| the process and sometimes |  |  |
| students are not self-checking. |  |  |$\quad$.

## Item 10 - Feedback is frequently focused on mathematical concepts or strategies.

 This item evaluates the focus of feedback. Feedback may include specific information about reasoning, processes, or calculations. Feedback can take the form of correction, suggestion, prompting, cueing or reinforcing and affirming. Opportunities for feedback on concepts or strategies may occur with practice of new tasks, after an error, or reinforcing correct reasoning. When misconceptions arise, feedback should identify the incorrect reasoning, clarify the correct reasoning, and reinforce students' use of the correct reasoning.| Implemented-3 | Partially Implemented-2 | Not Implemented-1 |
| :---: | :---: | :---: |
| Feedback is frequently focused on mathematical concepts or strategies. | Feedback is sometimes focused on mathematical concepts or strategies. | There is no feedback, OR feedback is not focused on mathematical concepts or strategies. |
| Examples: <br> - When the student provides an answer of 8 for $6 \times 2$, the teacher gives corrective feedback with a model and describes it as groups and size of group. <br> - Students are successful with the tasks. The teacher takes several opportunities to give informative feedback such as, "Good. You remembered to put the digits in their correct place value slot." | Examples: <br> - Feedback is specific and informative when students make a mistake, but affirmative feedback is not specific. For example, a student provides the answer of 8 for $6 \times 2$. The teacher gives corrective feedback. When the student is successful with the next problem of $4 \times 3$, the teacher' just says, "You got it." The teacher could say, "Good. You thought multiplication instead of addition." | Examples: <br> - The teacher consistently only tells students if they are right or wrong. If they are wrong, the teacher says, "Try again." <br> - Students are successful with the tasks. The teacher only tells students, "Good job." |

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